Passive Earth Pressure Coefficients in the Presence of Seepage Flow

Abdul Hamid SOUBRA — Maître de Conférences, E.N.S.A.I.S., Strasbourg, France
Richard KASTNER — Professor, I.N.S.A., Lyon, France
Jean-Luc HERITIER — Student Engineer, E.N.S.A.I.S., Strasbourg, France
Abdessamad BENMANSOUR — Research student, E.N.S.A.I.S., Strasbourg, France

Abstract

This paper describes an upper-bound method in limit analysis for calculating the passive earth pressure coefficients in the presence of seepage flow. The mechanism used is a translational one consisting of a log-sandwich. The numerical results obtained are presented and compared with other authors results.

Introduction

For deep retaining structures such as single sheet piles or double-walled cofferdams, where the excavation level is well below the ground water table, the seepage flow takes place around these structures.

The risk due to the seepage flow is traditionally taken into account by practising engineers in considering the piping and heaving risk: the calculation of heaving risk is usually made by considering a rectangular mechanism adjacent to the sheet piling wall (Terzaghi 1943). The vertical force equilibrium of this soil mass is then considered by neglecting the soil-structure force represented by the resultant of the passive earth pressures.

Kastner (1982), based on laboratory model tests, has shown that failure of the sheet piling structures is not only due to the heaving phenomenon but it can be produced by the reduction of the passive earth pressures. So it is interesting to propose methods for the calculation of the passive earth pressures taking into account the seepage forces.

In a previous paper, Soubra and Kastner (1992), proposed a method based on a variational approach: the variational limit equilibrium method. This method is equivalent to the upper-bound method in limit analysis for a rotational log-spiral mechanism (Soubra 1989).

In this paper, we present a method which takes into consideration the seepage forces and which is based on the upper-bound method in limit analysis for a translational log-sandwich mechanism. So the solution presented in this paper is an upper-bound one for a perfectly plastic material obeying Hill's maximal work principle.

Presentation of the mechanism

As it has been mentioned in the preceeding section, a translational wall motion is considered in this paper. The following generally adopted assumptions are taken into consideration:

- The soil is cohesionless with zero surcharge loadings.
- The angle of friction at the soil-structure interface is constant and equal to δ. This assumption is compatible with the translational wall kinematic.

Chen (1975) have considered six translational mechanisms to calculate the passive earth pressures in the case of cohesionless soil without seepage flow. The numerical results obtained have shown that the best unfavorable mechanism in most cases is the log-sandwich one. In this paper we adopt the same mechanism to calculate the passive earth pressures in presence of seepage flow.

As it is well known, the upper-bound theorem states that the soil mass will collapse if there is any compatible pattern of plastic deformation for which the rate of work of the external loads exceeds the part of internal dissipation. Thus, equating the external rate of work of all external forces to the internal rate of dissipation of energy gives an upper-bound of the exact solution for an associated flow rule material.

The log-sandwich mechanism is kinematically admissible since it verifies all the kinematical and velocity boundary conditions for an associated flow rule material.

To show the influence of the seepage flow on the passive earth pressure coefficients, we will consider in the following section a practical simple case where the hydraulic head distribution is known analytically in the soil mass.

Case study

We consider a sheet pile whose penetration depth is equal to f and subject to a hydraulic head H as shown in figure (1). The soil is assumed to be homogeneous and isotropic with respect to the hydraulical caracteristics ($K_h/K_v=1$) where K_h and K_v represent the horizontal and vertical permeability coefficients respectively.

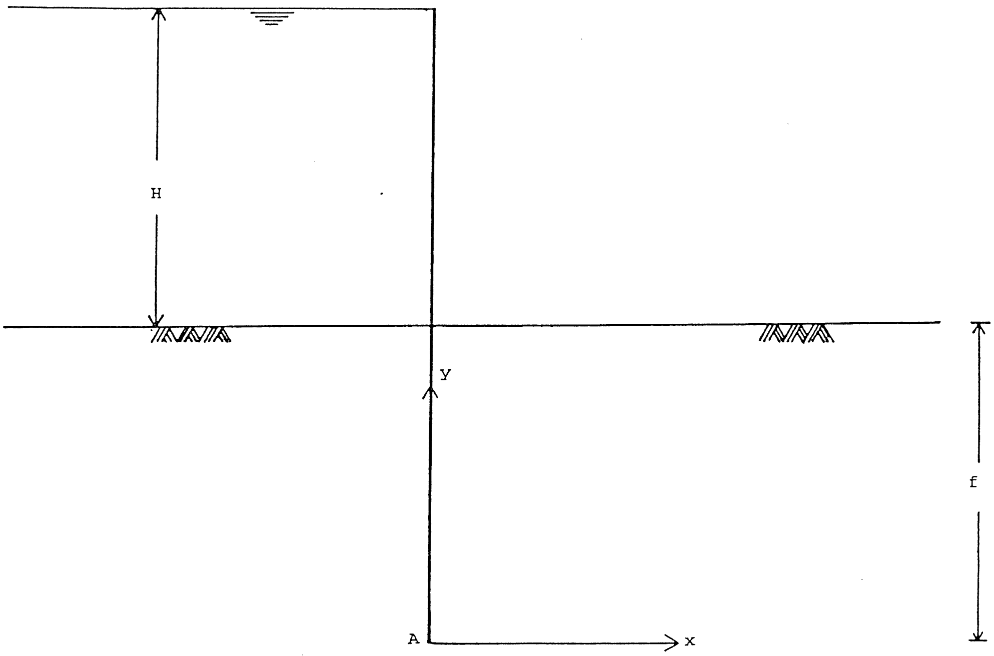


Fig 1 Case study

In this case, the relation which governs the hydraulic head at any point (x,y) in the coordinate system (Ax,Ay) shown in figure (1) is given by:

$$\frac{(f-y)^2}{f^2\cos^2\phi}-\frac{x^2}{f^2\sin^2\phi}=1 \qquad (1)$$

where

$$\phi=\pi.\left(\frac{h_s}{H}-\frac{1}{2}\right) \qquad (2)$$

From these equations, one can easily show that the pore water pressure at any point (x, y) in the downstream of the sheet pile is given by:

$$p=(h-y).\gamma_w=(h_s+f-y).\gamma_w \qquad (3)$$

where

$$h_s=\frac{H}{2}-\frac{H}{\pi}.\arcsin\sqrt{X} \qquad (4)$$

$$X=\frac{-y^2-x^2+2.f.y+\sqrt{(y^2-2.f.y+x^2)^2+4f^2x^2}}{2f^2} \qquad (5)$$

The mechanism shown in figure (2) is composed of a logarithmic spiral shearing zone OBC, sandwiched beetween two triangular rigid blocks OAB and OCD. The velocity field is shown in figure (3). As it was shown by Chen (1975), the velocity V(θ) is equal to $V_1.e^{\theta \tan\phi}$.

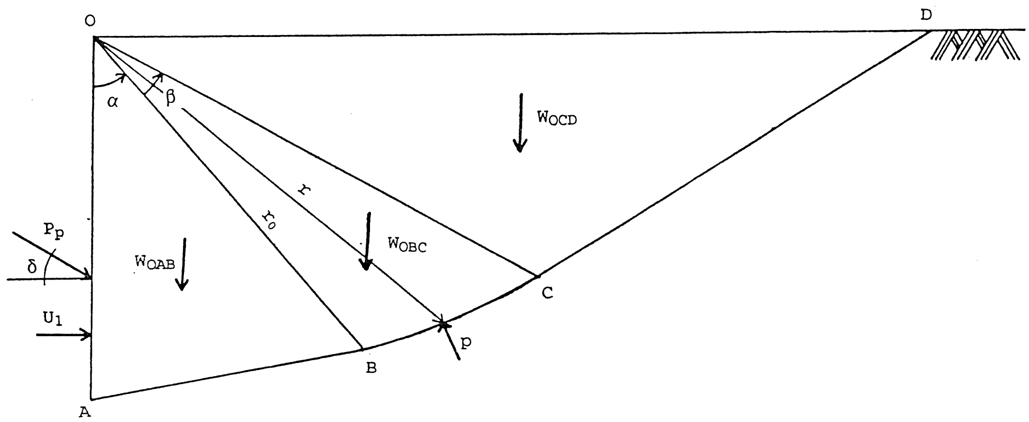


fig.2 Log-sandwich mechanism for passive earth pressure analysis in presence of seepage flow.

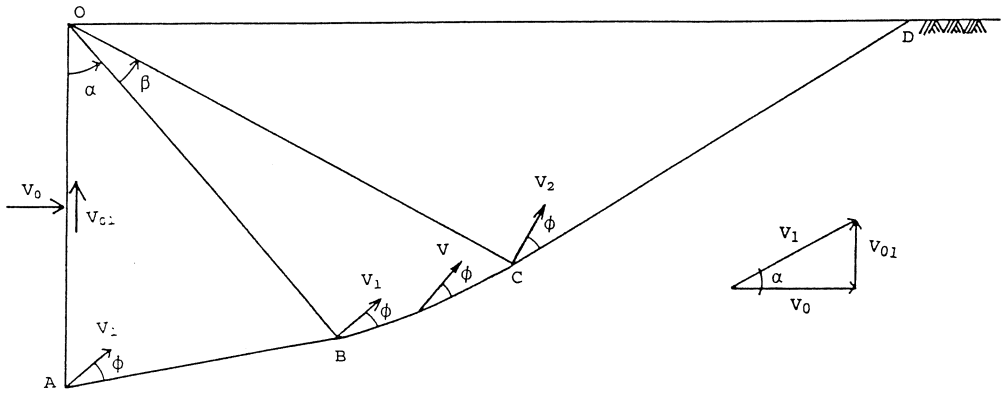


Fig.3 Velocity field

We will proceed now to the application of the limit theorem (the upper-bound technique) to the adopted mechanism by computation of the external rate of work and the internal energy dissipation.

External rate of work

The external rate of work is the external forces multiplied by the corresponding velocities. As shown in figure 2, the external forces consist of:

-the weight of blocks OAB,OBC,OCD
-the passive earth force Pp
-the pore water pressures along the slip surface ABCD
-the pore water pressures along the penetration depth for which the resultant is U_1.
-As the log-spiral shear zone is made of an infinity of rigid blocks with different velocities for each of them, we must

consider the rate of work of the pore water pressures radially in the log-spiral shear zone.

• *Rate of external work of the weight of OAB and OCD*

The rate of external work due to the self weight of OAB (or OCD) is simply the vertical component of velocity in the considered region multiplied by the weight of the region.

$$\Delta W_{OAB} = -\frac{1}{2}\frac{\gamma_{sat}.f^2.\sin^2\alpha.\cos(\alpha-\phi)}{\cos\phi}V_1 \tag{6}$$

$$\Delta W_{OCD} = -\gamma_{sat}.\frac{f^2}{2}.\frac{\cos^2(\alpha-\phi).\cos(\alpha+\beta).\sin(\alpha+\beta).e^{3\beta.\tan\phi}}{\cos\phi.\sin(\alpha+\beta-\phi)}.V_1 \tag{7}$$

• *Rate of work of the weight of OBC*

The elementary external work of the weight of the radial shear zone is simply the vertical component of the velocity multiplied by the elementary weight as follows

$$dW = -\frac{1}{2}.\gamma_{sat}.r^2.V.\sin(\alpha+\theta).d\theta = -\frac{1}{2}.\gamma_{sat}.r_0^{\,2}.e^{2\theta\tan\phi}.V_1.e^{\theta\tan\phi}.\sin(\alpha+\theta).d\theta$$

The total rate of external work is

$$\Delta W_{OBC} = -\frac{1}{2}.\gamma_{sat}.r_0^{\,2}.V_1\int_0^\beta e^{3\theta\tan\phi}.\sin(\alpha+\theta).d\theta \tag{8}$$

i.e.

$$\Delta W_{OBC} = -\frac{1}{2}.\frac{\gamma_{sat}.f^2.\cos^2(\alpha-\phi)}{\cos^2\phi.(1+9\tan^2\phi)}\Big\{\sin\alpha\big[-3\tan\phi+(3\tan\phi.\cos\beta+\sin\beta).e^{3\beta\tan\phi}\big]$$
$$+\cos\alpha.\big[1+(3.\tan\phi.\sin\beta-\cos\beta).e^{3\beta\tan\phi}\big]\Big\}.V_1 \tag{9}$$

• *Rate of work of the passive earth force*

The external work done by the component of the resultant wall load P_p moving with the velocity V_0 is given by

$$\Delta WP_{pn} = P_{pn}.\cos\alpha.V_1 \tag{10}$$

where P_{pn} is the P_p's normal component to the sheet piling wall.

• *Rate of work of the pore water pressures along the penetration depth*

U_1 is the resultant force of pore water pressures along the penetration depth f

$$U_1 = \int_0^f p.dy = \int_0^f (h-y).\gamma_w.dy = \int_0^f (h_s + f - y).\gamma_w.dy \tag{11}$$

thus, the rate of work of this force along the penetration depth is

$$\Delta WU_1 = \left[\frac{\gamma_w.f^2}{2} + \gamma_w.f.H\left(\frac{1}{2} - \frac{1}{\pi}\right)\right].\cos\alpha.V_1 \tag{12}$$

- *Rate of work of pore water pressures along AB and CD*

The elementary rate of work is the elementary force (p.ds) times the component of velocity normal to the corresponding line AB or CD, thus the rate of work is

$$\Delta WP_{AB} = \int_{AB} p.ds.V_1.\sin\phi \tag{13}$$

$$\Delta WP_{CD} = \int_{CD} p.ds.V.\sin\phi \tag{14}$$

In these equations p is a complex function of x and y. The integral was performed numerically using the Gaussian quadrature method.

- *Rate of work of pore water pressures along BC*

The pore water pressures act normally to the log-spiral surface, so, the elementary external work due to this pressure consist in multiplying the force (p.ds) by the normal component of velocity (V.sin ϕ) so the external rate of work is

$$\Delta WP_{BC} = \int_{BC} p.ds.V.\sin\phi = r_0.\tan\phi.\int_0^\beta p.e^{2\theta\tan\phi}.d\theta.V_1 \tag{15}$$

where

$$r_0 = OB = \frac{f.\cos(\alpha - \phi)}{\cos\phi} \tag{16}$$

- *Rate of work of pore water pressures in the log-spiral shear zone.*

The log-spiral is made of an infinity of rigid blocks with different velocities. Thus there is an external rate of work due to the pore water pressures in the log-spiral shear zone.
Two adjacent infinitly small rigid blocks are considered in figure (4).

The external rate of work done by the pore water pressures in the radial shearing zone of the log-spiral can be computed by summing first along the radius r, the product of the block velocity with the elementary force (p.dr) and finally by summing this result over the region β.

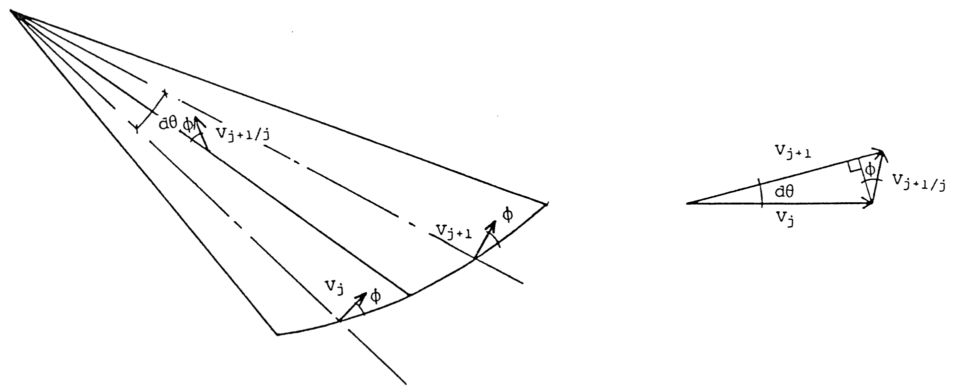


fig.4 Two adjacent rigid blocks of the shearing zone and the corresponding velocity hodograph.

The rate of work is then

$$\Delta WP_r = \int_0^{\beta} \left(\int_0^{r} p.V_L e^{\theta \tan\phi}.\tan\phi.dr \right).d\theta \qquad (17)$$

As it has been mentioned before, this double integral was made by using the Gaussian quadrature method.

Finally, the total external rate of work is computed by summing all the rates presented.

$$\Delta We = \Delta W_{OAB} + \Delta W_{OCD} + \Delta W_{OBC} + \Delta WP_{pn} + \Delta WU_1 + \Delta WP_{AB} + \Delta WP_{CD} + \Delta WP_{BC} + \Delta WP_r \qquad (18)$$

We will pass now to the calculation of the internal energy dissipation.

Internal energy dissipation

The internal energy dissipation takes place along velocity discontinuities surfaces such as AB, BC, CD and along the radial lines in the shear zone OBC. Notice that in this paper, the soil is assumed to be cohesionless. So, the internal energy dissipation along these surfaces is vanishing. Notice howhever, that the tangential component of the wall force produce an internal energy dissipation due to the friction along the soil structure.

$$\Delta W_i = Pp_n.\tan\delta.V_{01} = Pp_n.\tan\delta.\sin\alpha.V_1 \qquad (19)$$

<u>Minimization procedure</u>

By equating the external rate of work ΔW_e to the internal rate of dissipation of energy ΔW_i, one gets the expression of the passive earth force in terms of α and β-angles, the parameters describing the log-sandwich mechanism. The α and β-angles for which the P_p-value is minimun determine the most critical sliding surface. A computer program has been developed for assessing the passive earth force and the corresponding critical slip surface.

Numerical results

To show the influence of the seepage flow on the passive earth pressures, we present graphically (fig.5) the variation of the K_{pe}-value with the ϕ-value for different values of the non-dimensional parameter H/f (H/f= 0, 0.5 , 1 , 1.5 , 2 , 2.5) when γ_{sat}/γ_w =2 and for δ/ϕ = -2/3 . From this figure one can easily see that the K_{pe}-value increases with the ϕ-increase and decreases with the H/f-increase. For example, the reduction is about 60% when H/f increases from zero to 2.5 for ϕ =40°, δ/ϕ = -2/3 .

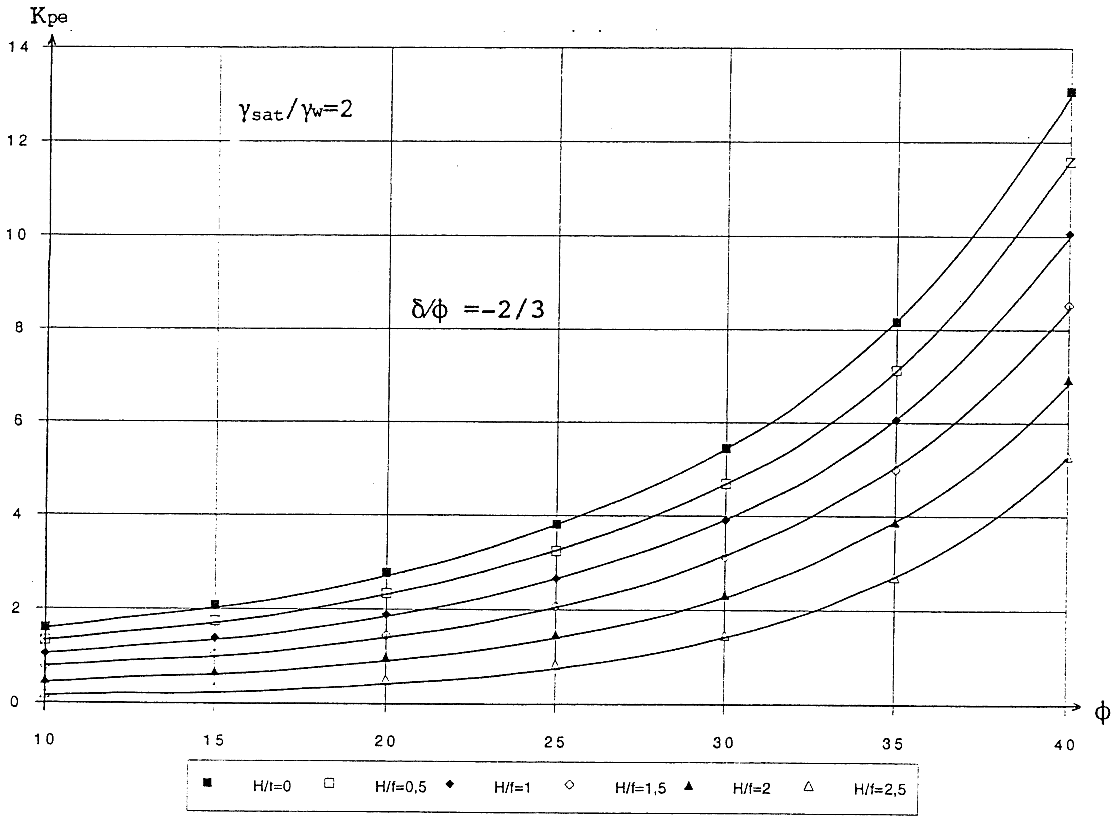


fig.5 Influence of hydraulic head on K_{pe}-values

To provide the engineer the K_{pe}-values for different soil caracteristics (ϕ, δ/ϕ) and for different values of H/f, we present in table (1a ,1b ,1c ,1d ,1e) the passive earth pressure coefficients K_{pe} to be used in the case of presence of seepage flow.

To show the influence of the δ/ϕ-value on the passive earth pressure coefficients for a given ϕ-value, we show in figure (6) the variation of the passive earth pressure coefficient with H/f for ϕ=30° and for 3 different values of δ/ϕ (δ/ϕ = -1/3 , -1/2 , -2/3).

φ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,42	1,17	0,92	0,67	0,42	0,13
15	1,7	1,41	1,11	0,82	0,52	0,2
20	2,04	1,7	1,35	1	0,65	0,28
25	2,46	2,06	1,65	1,24	0,83	0,39
30	3	2,52	2,04	1,56	1,06	0,54
35	3,69	3,12	2,55	1,97	1,38	0,75
40	4,6	3,92	3,23	2,53	1,82	1,06

(1a) $\delta/\phi=0$

φ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,52	1,25	1	0,72	0,46	0,18
15	1,89	1,57	1,25	0,93	0,6	0,26
20	2,39	2	1,61	1,21	0,81	0,38
25	3,08	2,6	2,11	1,62	1,11	0,58
30	4,06	3,45	2,83	2,21	1,57	0,89
35	5,49	4,71	3,93	3,13	2,3	1,43
40	7,72	6,7	5,66	4,61	3,53	2,38

(1b) $\delta/\phi=-1/3$

φ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,56	1,29	1,02	0,75	0,47	0,19
15	1,99	1,66	1,32	0,89	0,64	0,28
20	2,58	2,17	1,75	1,33	0,98	0,44
25	3,43	2,91	2,38	1,84	1,28	0,7
30	4,71	4,03	3,34	2,64	1,92	1,15
35	6,71	5,8	4,89	3,96	3	1,98
40	10,07	8,83	7,6	6,29	4,97	3,58

(1c) $\delta/\phi=-1/2$

φ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,6	1,33	1,05	0,77	0,49	0,2
15	2,08	1,74	1,39	1,04	0,69	0,31
20	2,78	2,34	1,9	1,45	0,99	0,5
25	3,81	3,24	2,67	2,08	1,48	0,84
30	5,44	4,68	3,92	3,13	2,33	1,48
35	8,17	7,12	6,06	4,99	3,88	2,71
40	13,08	11,58	10,06	8,52	6,94	5,29

(1d) $\delta/\phi=-2/3$

φ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,68	1,4	1,11	0,82	0,53	0,22
15	2,27	1,9	1,53	1,16	0,78	0,37
20	3,17	2,69	2,2	1,71	1,2	0,66
25	4,62	3,97	3,31	2,64	1,95	1,21
30	7,1	6,19	5,26	4,32	3,36	2,34
35	11,67	10,34	8,99	7,17	6,22	4,76
40	20,91	18,85	16,77	14,66	12,51	10,31

(1e) $\delta/\phi=-1$

Table 1: K_{pe}-values for different ϕ, δ/ϕ and H/f values (Translational mechanism)

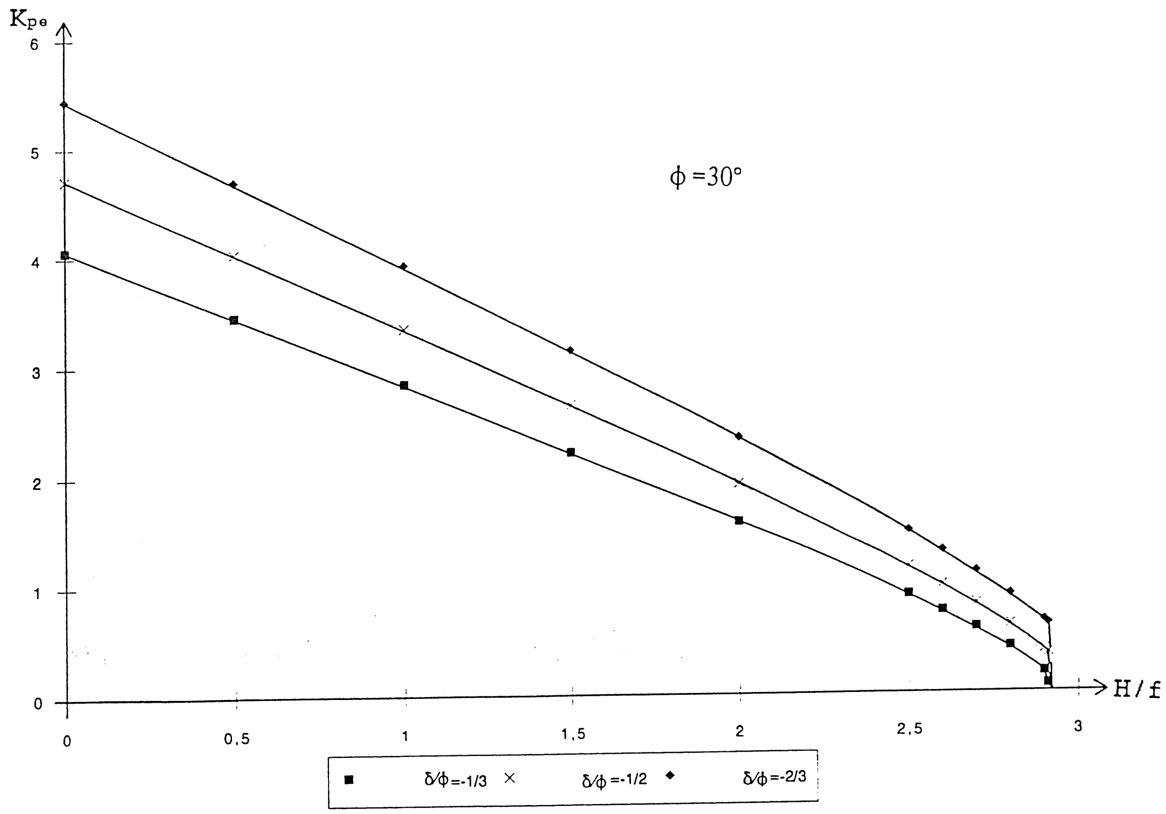


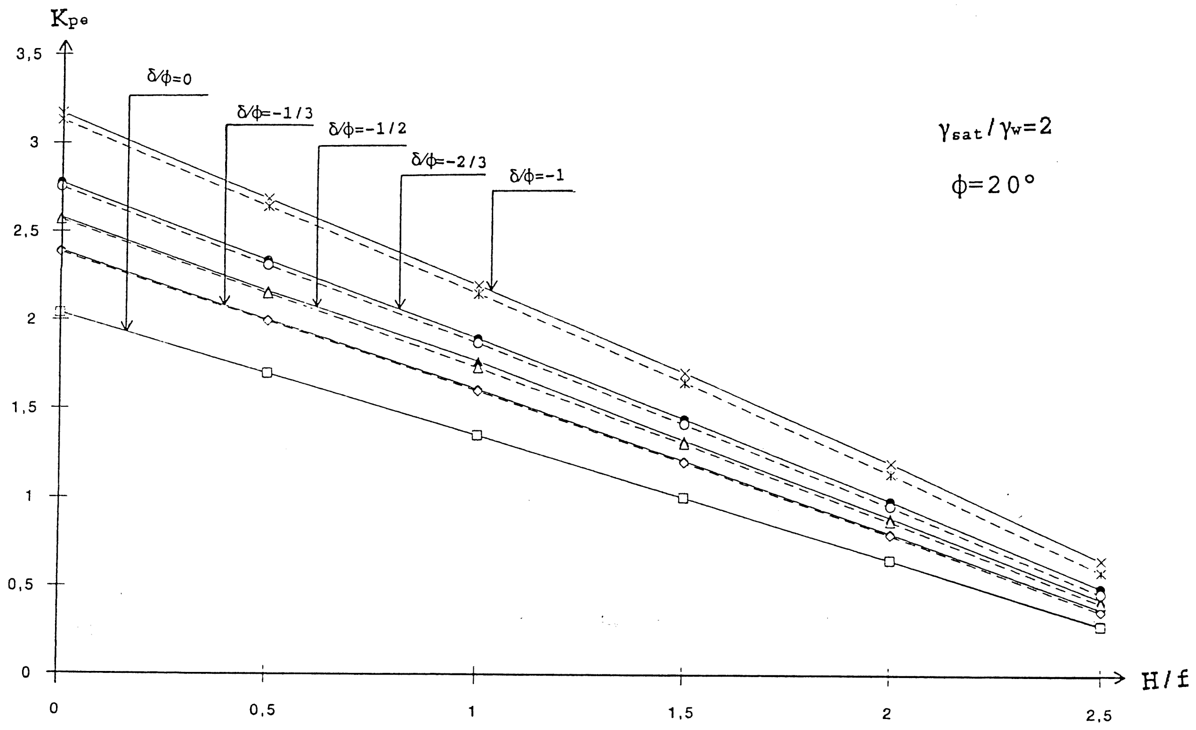
Fig.6 Variation of the K_{pe}-value with H/f.

It is easy to see that for zero K_{pe}-value, the corresponding H/f-value is the same for different δ/ϕ-values. This means that the angle of friction at the soil-structure interface has no effect on the H/f-value causing failure by heaving. This fact can be explained as follows: when the passive force is vanishing, there is no interaction at the soil-structure interface and we have the traditional heaving phenomenon. However, for small values of H/f for which the passive earth force do not vanish, this passive earth force depends on δ-value. Notice that Soubra et al (1992) have shown, in considering the rotational log-spiral mechanism, that the same so-mentioned phenomenon occurs i.e. for high H/f-values corresponding to failure by heaving, the δ/ϕ-value has no-effect on the K_{pe}-value.

We will pass now to the comparison of the present results with those of Soubra et al (1992).

Comparison with other results:

The results obtained in this paper are upper-bounds to the real values of the passive earth pressure coefficients for an associated flow rule material. The obtained values for the log-sandwich slip surface are the best ones for a translational mechanism. The numerical results of the passive earth pressure coefficients as obtained with the rotational mechanism (Soubra et al 1992) are compared with the above mentioned results. This will be done in figure (7) where we show the variation of the passive earth pressure coefficients with the non dimensional parameter H/f (H/f= 0, 0.5, 1, 1.5, 2, 2.5) for different values of δ/ϕ (δ/ϕ= 0, -1/3, -1/2 ,-2/3, -1) when γ_{sat}/γ_w=2 and ϕ= 40°, 20°.



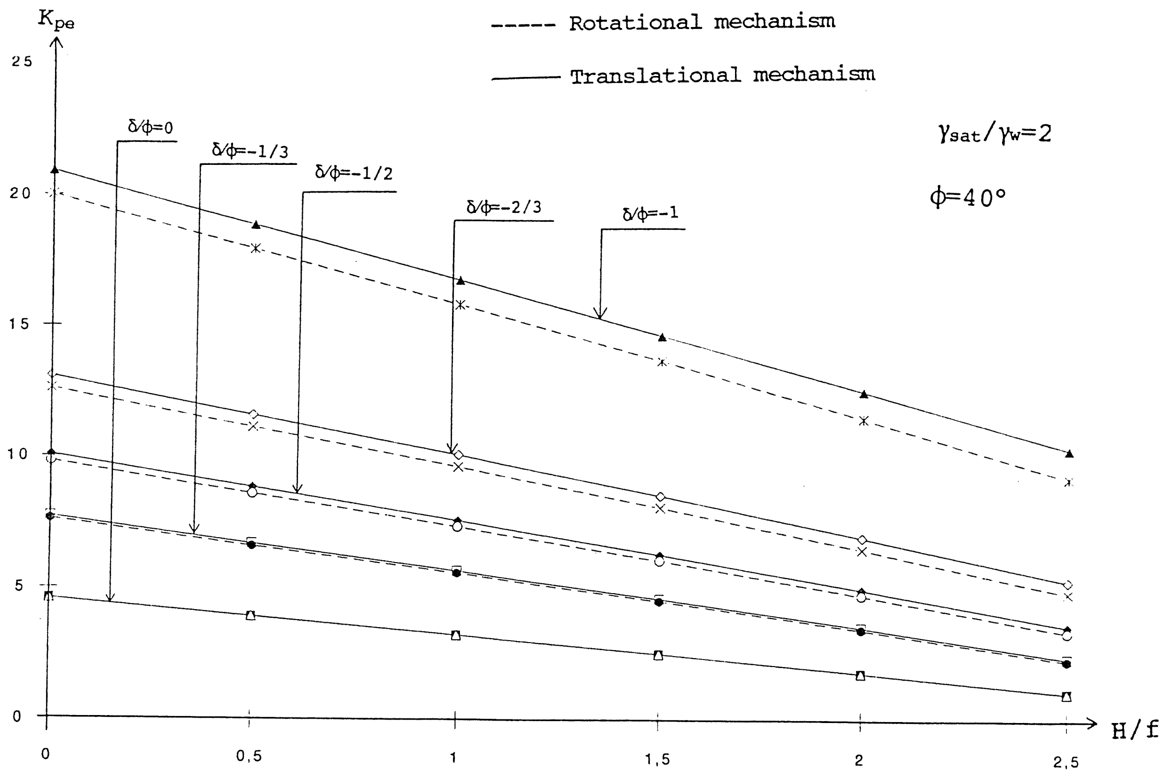


fig 7 Variation of the K_{pe}-value with H/f for ϕ=20° and ϕ=40°

From these figures, we can easily see, that the rotational mechanism gives smaller values of K_{pe} than the log-sandwich one. However, the difference is small in all cases of ϕ, δ and H/f. Moreover, the absolute difference between the K_{pe}-values of the two mechanisms increases with δ/ϕ-increase and is constant whatever the value of H/f. In fact the K_{pe}-values must be chosen as a function of the mechanism: thus, in order to provide the designer with the K_{pe}-values for the two mechanisms, we present in table (2a, 2b, 2c, 2d, 2e) the K_{pe}-values for the rotational mechanism.

ϕ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,42	1,17	0,92	0,67	0,42	0,13
15	1,7	1,41	1,11	0,82	0,52	0,2
20	2,04	1,7	1,35	1	0,65	0,28
25	2,46	2,06	1,65	1,24	0,83	0,39
30	3	2,52	2,04	1,56	1,06	0,54
35	3,69	3,12	2,55	1,97	1,38	0,75
40	4,6	3,92	3,23	2,53	1,82	1,06

(2a) $\delta/\phi=0$

ϕ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,51	1,25	0,98	0,72	0,44	0,15
15	1,89	1,57	1,25	0,92	0,59	0,24
20	2,38	1,99	1,6	1,2	0,8	0,37
25	3,07	2,58	2,09	1,6	1,09	0,56
30	4,03	3,42	2,81	2,18	1,54	0,87
35	5,44	4,66	3,87	3,08	2,26	1,39
40	7,62	6,6	5,57	4,52	3,44	2,3

(2b) $\delta/\phi=-1/3$

ϕ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,56	1,29	1,01	0,74	0,46	0,15
15	1,98	1,65	1,31	0,97	0,63	0,26
20	2,57	2,15	1,73	1,31	0,87	0,41
25	3,4	2,88	2,35	1,81	1,25	0,66
30	4,65	3,97	3,28	2,58	1,86	1,09
35	6,59	5,69	4,78	3,85	2,89	1,87
40	9,81	8,58	734	6,06	4,75	3,36

(2c) $\delta/\phi=-1/2$

ϕ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,6	1,32	1,04	0,76	0,48	0,16
15	2,07	1,73	1,38	1,03	0,67	0,28
20	2,75	2,31	1,87	1,42	0,96	0,47
25	3,76	3,19	2,62	2,03	1,43	0,79
30	5,34	4,58	3,81	3,03	2,23	1,37
35	7,95	6,91	5,86	4,78	3,68	2,5
40	12,59	11,11	9,61	8,08	6,05	4,84

(2d) $\delta/\phi=-2/3$

φ	H/f=0	H/f=0,5	H/f=1	H/f=1,5	H/f=2	H/f=2,5
10	1,67	1,39	1,1	0,8	0,58	0,18
15	2,25	1,88	1,51	1,13	0,74	0,33
20	3,13	2,64	2,15	1,65	1,14	0,59
25	4,54	3,88	3,21	2,53	1,84	1,09
30	6,93	6	5,07	4,12	3,14	2,09
35	11,3	9,95	8,58	7,19	5,76	4,25
40	20,01	17,93	15,83	13,68	11,48	9,19

(2e) $\delta/\phi=-1$

Table 2: K_{pe}-values for different ϕ, δ/ϕ and H/f values (Rotational mechanism)

After having compared the K_{pe}-values, we are going now to present the critical slip surfaces.

Critical slip surfaces

Figure (8) shows the critical slip surfaces as obtained by the translational and rotational mechanism for $\phi=40°$, $\delta=20°$, H/f=2. Notice that the log-sandwich mechanism is more extended than the log-spiral one and it gives greater K_p-values. It is also shown in the same figure, the critical slip surfaces obtained by the present log-sandwich mechanism in the case ($\phi=40°$, $\delta=20°$, H/f=0, 2)

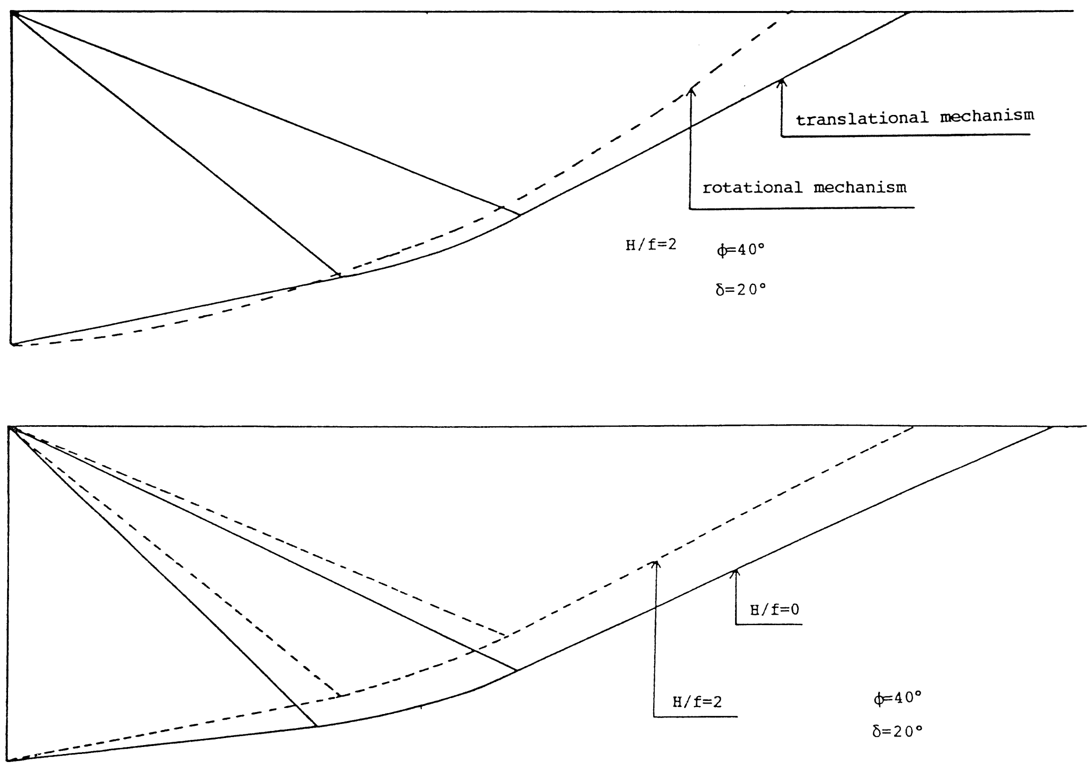


Fig.8 Critical slip surfaces

From figure (8), it is clear that the seepage flow shifts the sliding surface to less favorable positions. Thus, in addition to the reduction of the passive earth pressure coefficient, the critical sliding surface is also altered. This result has been shown by Soubra et al (1992) for the rotational mechanism.

Conclusion

A translational log-sandwich mechanism is used to calculate the passive earth pressure coefficients in presence of seepage flow. The results obtained are greater than those of Soubra et al (1992) for a rotational log-spiral mechanism. However the maximal absolute difference between the two mechanisms doesn't exceed 4% and the difference with the lower bound solution obtained by Lysmer (1970) is smaller than 5% in the case of $\phi=40°$, $\delta=20°$, $H/f=0$ available in litterature.

References

Kastner, R. (1982). Excavations profondes en site urbain: problèmes liés à la mise hors d'eau. Dimensionnement des soutènements butonnés. Thèse de Docteur ès Sciences, I.N.S.A. Lyon et Université Claude Bernard, Lyon I, 409p.

Lysmer, J. (1970). Limit analysis of plane problems in soil mechanics, J. Soil Mechanics & Foundation Division, ASCE, Vol. 96, N° SM4, Proc. paper, N° 7416, 1311-1334.

Soubra, A.H. (1989). Application de la méthode variationnelle au problème de détermination des pressions passives des terres. Influence des forces d'écoulement. Thèse de Doctorat, INSA Lyon, 200p.

Soubra, A.H., Kastner, R. (1992). Influence of seepage flow on the passive earth pressures. International Conference on retaining structures, Cambridge, 20-23 July, in press.

Terzaghi, H. (1943). Theoretical soil mechanics. New-York, Wiley.