Passive earth pressures using the upper–bound method in limit analysis

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SYNOPSIS

This paper describes a method for calculating the passive earth pressures acting on retaining structures by the upper bound theorem in limit analysis. The failure mechanism used is a rotational log–spiral. The numerical results obtained in the case of a cohesionless soil are presented and compared with other authors results.

1. INTRODUCTION

The problem of the active and passive earth pressures has been widely studied in literature since Coulomb (1776). In this paper, we focus our study on the passive earth pressure problem. The extension of the present model to the active earth pressure can be easily obtained.

In fact, this problem of the passive earth pressure belongs to the stability problems in geotechnical engineering and it has been modelled by either the limit equilibrium methods (Coulomb 1776, Terzaghi 1943, Packshaw 1969, Shields & Tolunay 1972, 1973, Rahardjo & Fredlund 1984), the slip line methods (Caquot & Kérisel 1956, Sokolovski 1960) or the limit analysis methods (Lysmer 1970, Lee & Herington 1972, Chen & Rosenfarb 1973).

While the limit equilibrium methods are simple, the slip line methods are more complicated since they require the establishment of a stress field in the plastically deformed region. Notice however that these methods do not allow to know if the solution obtained is an upper or a lower bound one with respect to the exact solution. The limit analysis method is based on the limit theorems of Drucker et al (1952) and it is employed to obtain upper and lower bounds of the collapse load for stability problems in geotechnical engineering, using the upper and lower bound methods.

The lower–bound method in limit analysis has not been widely used in geotechnical engineering since it is so complex for obtaining the statically admissible stress field in the soil mass. However, due to the facility of establishment of kinematically admissible mechanisms, the upper–bound method has been used for many mechanisms especially by Chen (1975) who presented the solutions for stability problems in geotechnical engineering.

Chen (1975) have presented the solutions of the passive earth pressure problem by considering six translational mechanisms. In this paper, we present an upper–bound method in limit analysis for calculating the passive earth pressure coefficients. The mechanism chosen for the calculation scheme is a rotational one. It consists of a log–spiral slip surface as it will be shown in detail in the following sections.

2. HYPOTHESES AND COLLAPSE MECHANISM

For the problem of computation of the passive earth pressures, the following assumptions will be adopted here :

- The soil is cohesionless
- The wall is vertical and the backill is horizontal.
- The point of action of the resultant of the passive earth pressures is assumed to act at the bottom third of the wall. In fact, this assumption depends on wall kinematics.
- The angle of friction at the soil–structure interface is assumed to be constant and equal to δ.

Notice that the traditional well known method for calculating the passive earth pressures is that of Coulomb (1776). This method is a limit equilibrium method giving unsafe solutions since it highly overestimates the passive earth pressure coefficients: this fact is due to the a–priori assumptions concerning the shape of the slip surface.

In a previous paper, Soubra et al (1992) adopted the same approach presented by Baker & Garber (1978) for the bearing capacity problem, and showed that the shape of the mechanism giving the minimal value of the passive earth force is a rotational log–spiral one. The method used was a variational limit equilibrium method. The solution obtained is a rigorous one in the sense of the limit equilibrium methods since it verifies all the global static equilibrium equations for the soil mass at failure and it does not make assumptions concerning the shape of the slip surface and the normal stress distribution along this surface. Notice also that this variational limit equilibrium method is equivalent to the upper–bound method in limit analysis for a rotational log–spiral mechanism. We will present in the following section the detail of the upper–bound method.

3. THEORETICAL BACKGROUND

According to the upper–bound theorem in limit analysis, for a kinematically admissible velocity field, an upper–bound of the exact collapse load can be obtained by equating the power dissipated internally to the power expended by the external loads. A

kinematically admissible velocity field is one that satisfies the flow rule, the velocity boundary conditions, and compatibility. During plastic flow, power is assumed to be dissipated by plastic yielding of the soil mass, as well as by sliding along velocity

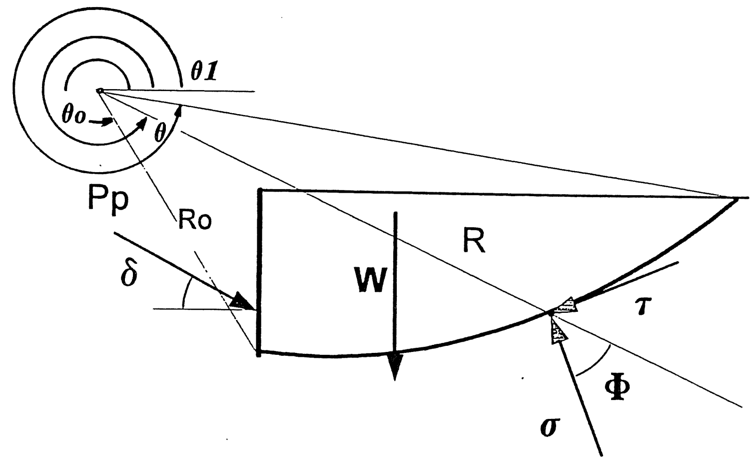

Figure 1 . Collapse mechanism

discontinuities where jumps in the normal and tangential velocities may occur.

In this paper, the mechanism chosen for the calculation scheme is a rotational log–spiral one (Figure 1), whose equation is given by

$R=Ro.e^{(\theta-\theta o)\tan\phi}$

This mechanism is assumed to be rigid, rotating with an angular velocity Ω around the centre of the log–spiral. Notice that this rotational mechanism has been used in limit analysis to solve different problems in soil mechanics such as the slope stability (Chen 1975, Anthoine 1990, De Buhan et al 1992) and the bearing capacity of foundations (Soubra 1992). This mechanism is kinematically admissible since it verifies all the kinematical constraints mentioned above.

In fact, this collapse mechanism depends on two parameters θ_o and θ_1. These angular parameters describe completely the log–spiral slip surface. Notice that, equating the external rate of all external forces to the internal energy dissipation for a rotational log–spiral mechanism is equivalent to the moment equilibrium equation for the same mechanism around the centre of the log–spiral surface: this was shown by Baker & Garber (1978) in the bearing capacity of foundations and by Soubra (1989) in the passive earth pressure problem.

As shown in Figure 1, the forces acting on the collapse mechanism are:

- The weight of the soil mass between the log–spiral slip surface and the ground surface.
- The passive earth force Pp which is inclined at δ to the normal of the wall face.
- The normal and tangential stresses acting along the slip surface. The soil is assumed to be cohesionless, so the moment of the resultant of the normal and tangential stresses is vanishing.

By writing the moment equilibrium equation for the soil mass and due to the assumption concerning the position of the resultant passive force Pp, one obtains the passive earth pressure coefficients Kp as a function of the two parameters θo and θ_1. The most critical Kp–value can be obtained by minimising with respect to θo and θ_1–angles shown in Figure 1. The θo and θ_1–angles at which the Kp–value is minimum determine the most critical slip surface. A computer FORTRAN program has been developed for assessing the minimal Kp–values and the corresponding critical slip surfaces. The minimisation procedure is based on Powell's quadratically convergent method.

4. NUMERICAL RESULTS

The numerical results obtained from the minimisation procedure are presented in Table 1 for the ϕ–value varying from 10° to 40° and for five different values of δ/ϕ (δ/ϕ=0, –1/3, –1/2, –2/3, –1). From these results, it is interesting to notice that for zero δ–value, i.e. in the case of a smooth wall, the passive earth pressure coefficient is the same as the one given by Rankine or Coulomb, i.e. $Kp=(1+\sin\phi)/(1-\sin\phi)$, which is the exact solution: the corresponding log–spiral slip surface degenerates to a planar surface inclined at $\pi/4-\phi/2$ to the horizontal.

Table 1 . Kp values for cohesionless soil

δ/Φ Φ	0	–1/3	–1/2	–2/3	–1
10	1.42	1.51	1.56	1.60	1.67
15	1.70	1.89	1.98	2.07	2.25
20	2.04	2.39	2.57	2.75	3.13
25	2.46	3.07	3.40	3.76	4.54
30	3.00	4.03	4.65	5.34	6.93
35	3.69	5.44	6.59	7.95	11.30
40	4.60	7.62	9.81	12.59	20.01

5. COMPARISONS AND DISCUSSIONS

To investigate the validity of the present theoretical analysis, it is generally required to compare the numerical results of the analysis with measurements from field or from laboratory model tests. Unfortunately, in the field there is complicated boundary conditions and failure takes place progressively. On the other hand, in the laboratory model tests, the comparison is difficult due to the uncertainties concerning the scale effect. Hence, the investigation of the present results will be made by the comparison with the theoretical solutions, especially those concerned with the limit analysis method allowing to bracket the collapse load between the upper and lower bound solutions. We will present in the following section, an analysis of the theoretical results available in literature, and will compare the present results with the currently accepted theoretical values.

5.1. Discussions on available theoretical results

We present in Table 2, the passive earth pressure coefficients obtained by many authors in the case (ϕ=40°, δ=20°). This case was chosen for two reasons: First, because it has been frequently studied by authors, and second, because in this case, there is big differences between authors results.

Notice that in the category of lower bound methods in limit analysis, Lysmer (1970) established an optimal lower bound solution for the passive earth pressure problem. Combining the local equilibrium equations with the Mohr–Coulomb yield criterion, and assuming that all stresses vary linearly within each triangular element of some mesh which covers the soil mass under study, the passive earth pressure problem has been transformed to a standard linear programming problem. Notice that the Mohr–Coulomb criterion is assumed linear: This assumption is on the safe side. On the other hand, Basudhar et al (1979) have used the same approach presented by Lysmer with certain variations to incorporate the non–linear no–yield condition constraints and thus, their problem was a non–linear programming problem.

As shown in Table 2, Basudhar et al (1979) have obtained a passive earth pressure coefficient which is smaller than that of Lysmer: This seems to be contradictory. Basudhar et al (1979) justify this result by the convergence criterion used in the analysis.

In the present paper, we will take the Lysmer's result as a basis for the lower bound methods, in order to compare with our results.

***Table 2. Passive earth pressure coefficients as obtained by many authors** (Φ=40°, δ=20°)*

Coulomb (1776)	11.77
Terzaghi (Packshaw, 1946)	10.82
Terzaghi (Shield & Tolunay, 1972)	9.66
Shield & Tolunay, 1973	8.30
Rahardjo & Fredlund (1984)	9.63
Habibagahi & Ghahramani (1979)	9.95
Caquot & Kérisel (1956)	9.60
Sokolovski (1960)	9.68
Lysmer (1970)	9.54
Basudhar & al (1979)	9.34
Lee & Herrington (1972)	9.30
Chen & Rosenfarb (1975)	10.1
Present solution	9.81

5.2. Comparisons

To compare the present results with the upper–bound solutions available in literature, Chen & Rosenfarb (1973) presented six mechanisms for which the log–sandwich gives the smallest values of the passive earth pressure coefficients in most cases.

Hence, the exact solution for an associated flow rule material must be between the ones of Lysmer and Chen &Rosenfarb in the case of the log–sandwich mechanism.

In fact our result for $\phi=40°$ and $\delta=20°$ is Kp=9.81 which is smaller than the smallest upper–bound solution (Kp=10.1) and higher than the lower–bound solution of Lysmer (Kp=9.54). The difference with the lower bound solution is thus smaller than 3% which indicates that the present solution is very close to the exact solution for an associated flow rule material obeying Hill's maximal work principle.

We will pass now to the passive earth

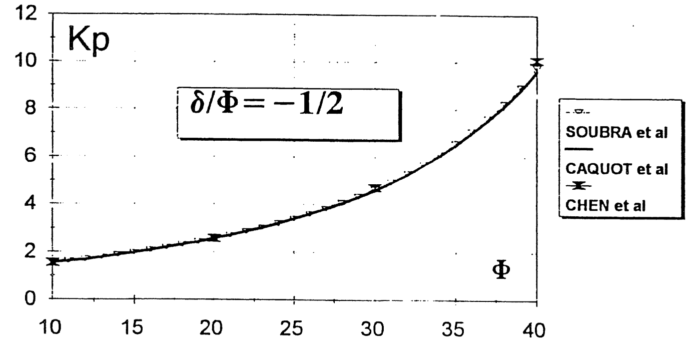

Figure 2. Comparaison with the results of Caquot & al and Chen & al ($\delta/\phi = -1/2$)

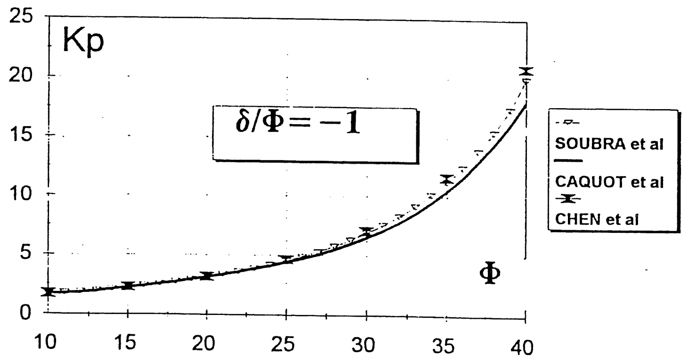

Figure 3. Comparison with the results of Caquot & al and Chen & al ($\delta/\phi = -1$)

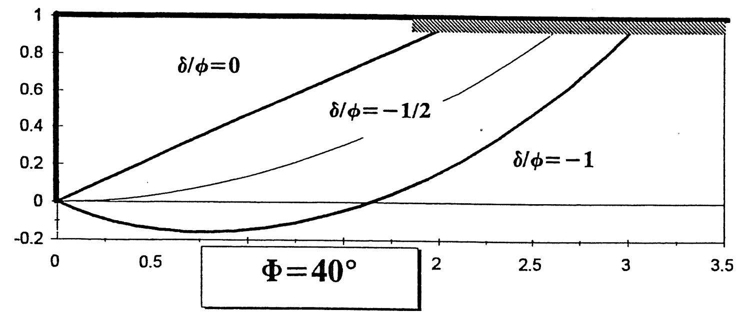

Figure 4. Critical slip surfaces for $\phi = 40°$

pressure coefficients obtained by Sokolovski (1960) and Caquot & Kérisel (1956) using the slip line method: The Kp–value obtained by these authors are equally presented in Table 2. In fact, the corresponding passive earth pressure coefficients lye in the range between the best lower and upper bound solutions given by the limit analysis method. Remember here that these methods give the currently accepted theoretical values.

Finally, we will compare the present results with the ones obtained by the limit equilibrium methods: Some authors (Coulomb 1776) give a value which is higher than the best upper–bound and others (Shields & Tolunay 1973) give a Kp–value which is smaller than the best lower bound solution. Hence, as it is well known, the values obtained by the limit equilibrium methods are approximate and we can not say if they are upper or lower bounds to the exact solutions for an associated flow rule material.

To present graphically the comparison between authors results, we present in Figures 2 and 3 the variation of the Kp–value with ϕ for two different values of δ/ϕ ($\delta/\phi=-1/2, -1$) for the present solution and for the ones given by Chen & Rosenfarb (1973) and Caquot & Kérisel (1956). It is easy to see that the present solution is close to the exact one for an associated flow rule material since it is bracketed by the one of Chen & Rosenfarb (1973) giving the best upper–bound solution and the one of Caquot & Kérisel (1956) which can be assumed as a lower bound solution.

We present in Figure 4 the critical slip surfaces as obtained by the present model for the case ($\phi=40°$, $\delta=0$, $\delta = -20$ and $\delta = -40°$). From this figure, it is clear that the slip surface approaches a planar surface when δ/ϕ decreases : when $\delta/\phi = 0$, we find the planar slip surface given by the Coulomb or Rankine mechanisms.

6. CONCLUSION

The upper-bound method in limit analysis for a rotational log-spiral mechanism has been developed for calculating the passive earth pressure coefficients. The values obtained by the present upper bound method are smaller than the smallest ones obtained by the same method available in the literature. The results obtained are in good agreement with the currently theoretical values bracketed between the best upper and lower bound values given by the limit analysis method.

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