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Bearing capacity in seismic areas by a variational approach

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SUMMARY

This paper describes a pseudo-static method for evaluating the seismic bearing capacity of a strip footing: Based on a variational approach this method is a rigorous one in the framework of the limit equilibrium methods since it does not make assumptions concerning the slip surface and the normal stress distribution along this surface. Some numerical results show the effect of the earthquake loadings on the critical slip surface and the corresponding normal stress distribution along this surface.

NOTATION

The following symbols are used in this paper:

B = integration constant
B_0 = breadth of the footing
H = Lagrange's intermediate functional
K_h = horizontal seismic coefficient
P = foundation load
W = weight of the soil mass
c = cohesion
r_0 = initial ray of the log-spiral
r = ray of the log-spiral mechanism
q = ultimate bearing capacity
(r,θ) = polar coordinate system
s = arc length along y(x)
(x,y) = cartesian coordinate system
x_1 = end point of the slip surface y(x)
y(x) = equation of the slip surface
ϕ = angle of internal friction
γ = unit weight of the soil
$\sigma(x)$, $\tau(x)$ = normal and tangential stress along y(x)

1. INTRODUCTION

The traditional method for evaluating the effect of an earthquake load on the stability of a soil-foundation system is the so-called "pseudo-static method". This method continues to be used by consulting geotechnical engineers because it is required by the building codes: it is easier and less costly to apply and it gives satisfactory results. This method will continue to be popular until an alternative method can be shown to be a more reasonable approach.

In this method, the inertia force is treated as an equivalent concentrated force (pseudo-static force) applied at the center of gravity of the structure (Meyerhof [1] and Shinohara et al [2]). However, this approach is not realistic since the soil mass is also affected by earthquake loadings. Recently, Sarma and Iossifelis [3] have considered this problem of the ultimate bearing capacity of strip footings in seismic areas by considering the inertial forces on both the soil and the structure.

Usually, the pseudo-static analysis is associated to the limit equilibrium methods: In these methods, two assumptions are usually made concerning the shape of the slip surface and the normal stress distribution along this surface. The so-mentioned a priori assumptions give approximate solutions for the bearing capacity problem.

In this paper, we present a variational approach applied to the limit equilibrium method. As it was shown by Garber and Baker [4] in the case of no seismic loadings, this approach allows to get the slip surface (kinematic function) and the normal stress distribution (static function) along this surface.

Finally, numerical results are presented: These results show the effect of the seismic loading on the critical slip surface and the corresponding normal stress distribution along this surface.

2. VARIATIONAL LIMIT EQUILIBRIUM METHOD

As it is generally known, the analysis of a stability problem in geotechnical engineering by a limit equilibrium method consists in considering a soil mass in a state of limit equilibrium: This state of limit equilibrium can be described by the following conditions:

-The three limiting equilibrium equations of the soil mass are satisfied.
-The Mohr-Coulomb criterion must be satisfied along the slip surface.

The equilibrium equations requires knowledge of both the shape of the slip surface (circle, log-spiral, log-sandwich, ect...) and the normal stress distribution along this surface. Traditionally, many assumptions are made concerning the two unknowns functions. Also, in the majority of the traditional limit equilibrium methods, the three equations of static equilibrium are not completely satisfied.

To get a rigourous solution in the framework of limit equilibrium methods, we present in this paper a variational approach applied to the limit equilibrium method. This method allows to get the kinematic and static functions giving the minimal foundation load and for which the three limiting equilibrium equations are satisfied.

The following assumptions are made in this analysis:

a. As was mentioned before, all inertias of the soil-structure system are considered in this analysis.
b. The earthquake acceleration for both the soil and the structure is assumed to be the same: Only the horizontal seismic coefficient k_h is considered in this analysis, the vertical seismic coefficient is often disregarded. Notice that the choice of the seismic coefficient is completely emperical (Seed [5], [6], [7]).
c. The earthquake load on the structure is represented by the base shear load acting at the foundation level and an eccentricity for the vertical foundation load.
d. A one sided failure is assumed to occur along the surface y(x) as shown in figure (1).
e. Only the reduction of the bearing capacity due to the increase in driving forces is investigated under seismic loading conditions. The shear strength of the soil is assumed to remain unaffected by the seismic loading.
f. The moment due to earthquake loading on the structure is not considered.

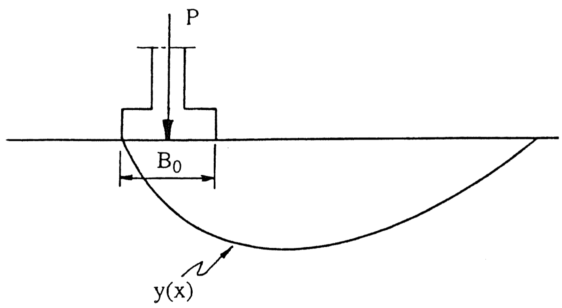

Fig. 1 : Slip surface for seismic bearing capacity analysis

2.1 Equilibrium equations

The external forces acting on the soil mass (Fig. 2) can be stated as follows:

a. The weight of the soil mass W and the corresponding horizontal inertial force $K_h.W$

b. The foundation load P and the corresponding inertial force $K_h.P$. This inertial force is represented by a base shear load on the foundation level.

c. The normal and shear stress distribution along the slip surface.

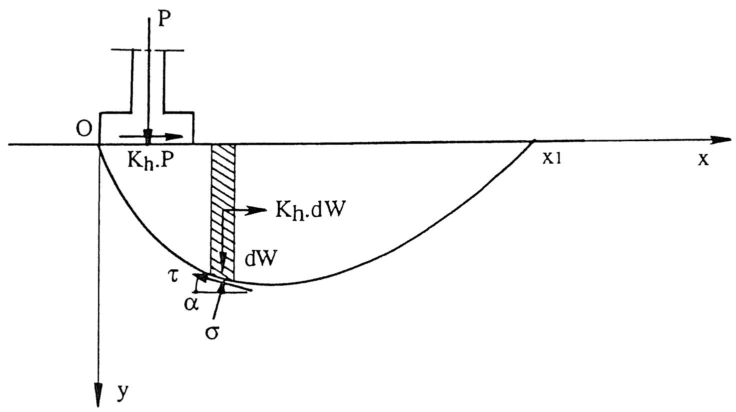

Fig. 2 : Free body diagram

Writing the three equilibrium equations for the soil mass (Fig. 2), one obtains:

$$K_h.P+K_h.\gamma\int_0^{x_1} y.dx+\int_s [\sigma.\sin\alpha-\tau.\cos\alpha]ds=0 \qquad (1)$$

$$P+\gamma\int_0^{x_1} y.dx-\int_s [\sigma.\cos\alpha+\tau.\sin\alpha]ds=0 \qquad (2)$$

$$P.B_0/2 + \gamma \int_0^{x_1} y[x - K_h.\frac{y}{2}]dx - \int_S [(\sigma.\sin\alpha - \tau.\cos\alpha)y + (\sigma.\cos\alpha + \tau.\sin\alpha)x]ds = 0 \qquad (3)$$

where equations (1) and (2) represent respectively the horizontal and vertical equilibrium equations and equation (3) represent the moment equation around O.

Combining these equations with the Mohr-Coulomb criterion, one obtains the three limiting equilibrium equations as follows:

$$K_h.P + \int_0^{x_1} [\sigma(y' - tg\phi) + K_h.\gamma.y - c]dx = 0 \qquad (4)$$

$$P - \int_0^{x_1} [\sigma(tg\phi.y' + 1) - \gamma.y + c.y']dx = 0 \qquad (5)$$

$$P.B_0/2 - \int_0^{x_1} [\sigma.x(1 + tg\phi.y') + \sigma.y(y' - tg\phi) - \gamma.y(x - K_h.\frac{y}{2}) - c(y - x.y')]dx = 0 \qquad (6)$$

Due to these equations, one can easly show that the foundation load P depends on two unknown functions y(x) and σ(x).

The rigorous seismic bearing capacity problem in the framework of limit equilibrium consists in finding the two functions y(x) and σ(x) giving the minimal failure load and for which the three limiting equilibrium equations are satisfied. This problem is in fact a variational problem of the isoperimetric type as follows:

$$\text{Min } P = \int_0^{x_1} [\sigma(y' - tg\phi) + K_h.\gamma.y - c]dx \qquad (7)$$

subject to:

$$\int_0^{x_1} [\sigma(tg\phi.y' + 1) - \gamma.y + c.y']dx = P \qquad (8)$$

$$\int_0^{x_1} [\sigma.x(1 + tg\phi.y') + \sigma.y(y' - tg\phi) - \gamma.y(x - K_h.\frac{y}{2}) - c(y - x.y')]dx = P.B_0/2 \qquad (9)$$

In such a problem the extremal functions giving the minimal failure load can be obtained by considering an intermediate functional H (Petrov [8]) defined as follows:

$$H = F + \lambda_1.G_1 + \lambda_2.G_2$$

where F(x,y,y',σ) can be obtained from equation (7) as follows:

$F=\sigma(y'-tg\phi)+K_h.\gamma.y-c$

The $G_i(x,y,y',\sigma)$ functions can be obtained from the two equations (8) and (9) as follows:

$G_1=\sigma(tg\phi.y'+1)-\gamma.y+c.y'$

$G_2=\sigma.x(1+tg\phi.y')+\sigma.y(y'-tg\phi)-\gamma.y(x-K_h.\frac{y}{2})-c(y-x.y')$

and λ_1 and λ_2 are the Lagrange's multipliers.

Finally, the two extremal functions y(x) and σ(x) must satisfy:

a. The system of Euler's differential equations for the intermediate functional H as follows:

$$\frac{\partial H}{\partial \sigma}=\frac{d}{dx}.\frac{\partial H}{\partial \sigma'} \qquad (10)$$

$$\frac{\partial H}{\partial y}=\frac{d}{dx}\frac{\partial H}{\partial y'} \qquad (11)$$

b. The integral constraints (the three limiting equilibrium equations)
c. The boundary conditions at the end points x_0 and x_1 as follows:
- At point O: (footing edge), we have $x_0=y_0=0$
- At point B: This point is free to move along the ground surface. For this point, the transversality condition must be satisfied in ordre to assure an extremal value for the functional.This condition is:

$$[H-y'.\frac{\partial H}{\partial y'}-\sigma'.\frac{\partial H}{\partial \sigma'}](x=x_1)\,.\,\delta x_1+[\frac{\partial H}{\partial \sigma'}](x=x_1).\,\delta\sigma_1+[\frac{\partial H}{\partial y'}](x=x_1).\delta y_1=0 \qquad (12)$$

2.2 Determination of potential kinematic and static functions

a. First Euler equation:

This equation is given by equation (10). Substituting the functional H into this equation, one obtains a first order differential equation of y only. Solving this differential equation, one obtains the equation of the slip surface as follows:

$r=r_0.\exp(\theta_0-\theta)tg\phi$

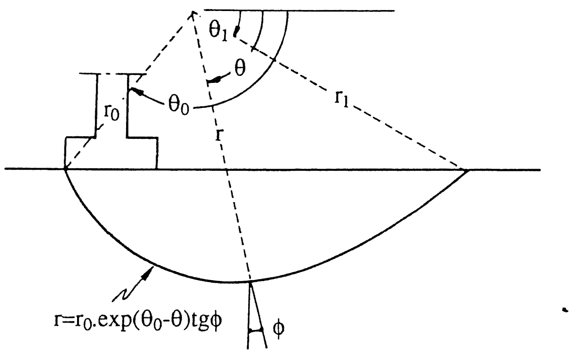

Fig. 3 : Log-spiral mechanism for seismic bearing capacity analysis

This equation is the slip surface equation in the case of a homogeneous soil of constant ϕ in the polar coordinate system as shown in figure(3).

Notice that the shape of the slip surface is independent of the seismic loading. Notice howewer that the seismic loads have an effect on the position of the critical slip surface corresponding to failure as it will be shown later.

b. Second Euler equation:

Substituting H into equation(11), one obtains a first ordre differential equation of σ only. The solution of this differential equation is given as follows

$$\sigma(\theta)=\gamma.r_0.A(\theta)-\frac{c}{tg\phi}+B.\exp(2\theta tg\phi)$$

$$\text{where : } A(\theta)=[\sin\theta(1+3K_h.tg\phi)+\cos\theta(K_h-3tg\phi)][\frac{\exp(\theta_0-\theta)tg\phi}{1+9tg^2\phi}]$$

$$B=[\sigma_1-\gamma.r_0.A(\theta_1)+\frac{c}{tg\phi}][\exp(-2\theta_1 tg\phi)]$$

c. Transversality condition:

Substituting H into equation(12), one obtains the normal stress value at point B as follows:

$$\sigma_1=\frac{-c.r_0.\sin\theta_0}{tg\phi.r_0.\sin\theta_0+r_1.\cos\theta_1-2r_0.\cos\theta_0}$$

2.3 Critical kinematic and static functions

Replacing the potential kinematic and static functions into the three equations of equilibrium (equations 4, 5, 6), one obtains a system of three non linear equations in three unknowns parameters P, θ_0 and θ_1.

A computer Pascal program has been developed to solve this system. The results obtained by the program allows to get the critical slip surface and the corresponding normal stress distribution. Also is given by the program is the minimal failure load P as it will be shown in the following section.

3. NUMERICAL RESULTS AND DISCUSSION

3.1 Influence of the seismic loadings on the ultimate bearing capacity

The numerical results obtained from the program concerning the variation of the ultimate bearing capacity with the the seismic loadings for different soil characteristics are presented in figure (4). For the purpose of convenient presentation, the two non-dimensional quantities $q/\gamma B_0$ and $c/\gamma B_0$ are employed.

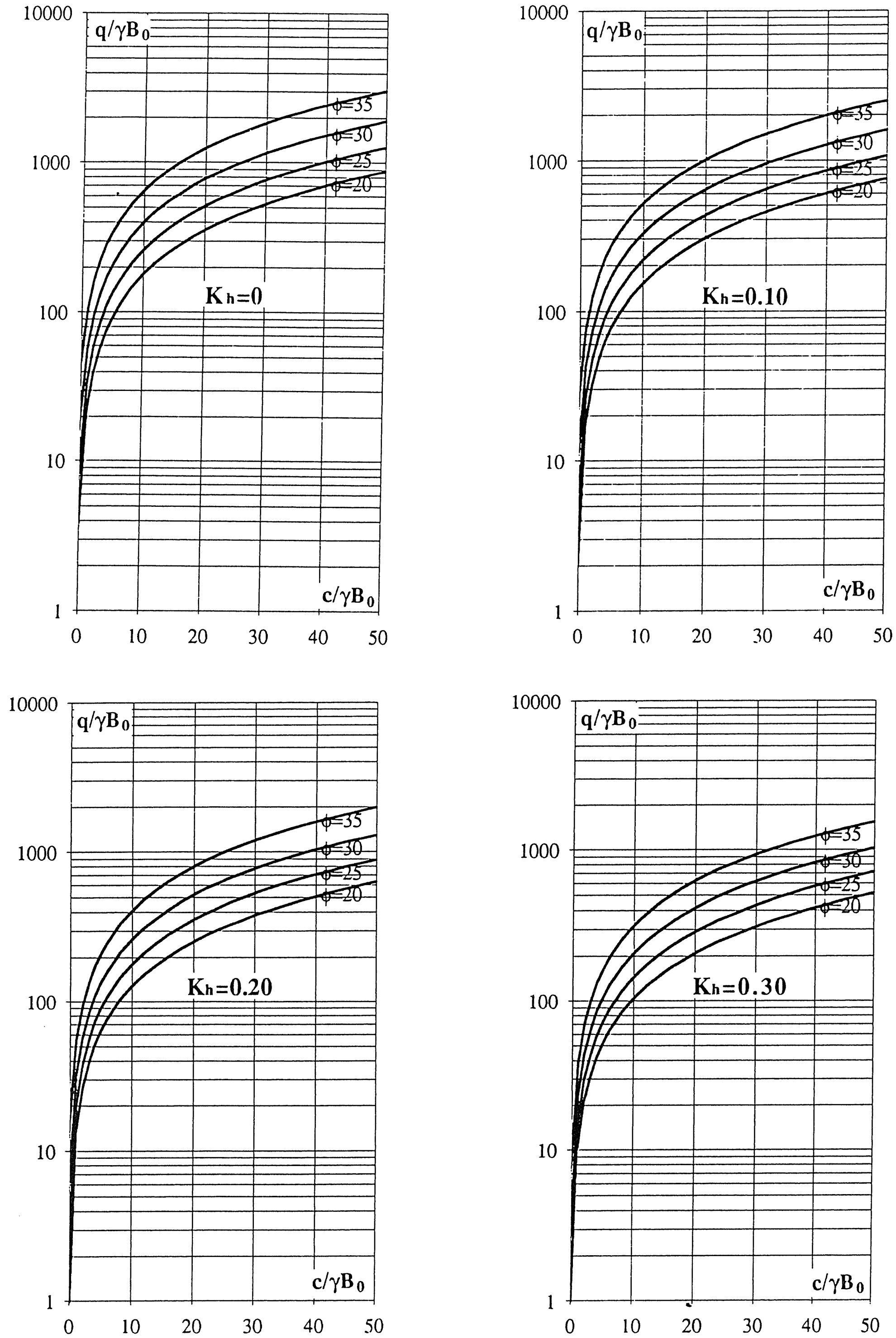

Fig. 4 : Effect of the seismic loadings on the ultimate bearing capacity

Notice that these results are identicals to those obtained by the upper-bound method in limit analysis for a rotational log-spiral mechanism (Soubra and Reynolds [9]) in the case (β=0°). This fact is due to the equivalence between the variational limit equilibrium method and the upper-bound method in limit analysis for a rotational log-spiral mechanism as it was shown by Soubra [10].

Due to figure (4), one can easily see that for ϕ=30°, $c/\gamma B_0=10$; the reduction in the $q/\gamma B_0$ value is about 48% when the horizontal seismic coefficient increases from zero to 0.3. Thus, the calculation of the ultimate bearing capacity taking into account the seismic loadings is of great interest in areas of high earthquake risks.

The comparison with other author's results has been given by Soubra and Reynolds [9]. This comparison has shown that there is good agreement with the experimental and theoretical results given by Saran and Agarwal [11].

Finally, notice that the present approach allows to get the critical normal stress distribution along the slip surface as it will be shown later.

3.2 Influence of the seismic loadings on the critical slip surface and the corresponding normal stress distribution

As it was seen before, the seismic loadings reduce the bearing capacity of a footing. It also shifts the position of the critical slip surface to less favorable positions. We present in figure (5) a numerical example showing the effect of the seismic loadings on the critical slip surface when (ϕ=30°, c=0, B_0=1m, γ=20kN/m^3 and k_h=0; 0.2). As it is shown, the slip surface becomes less extended for higher k_h-values.

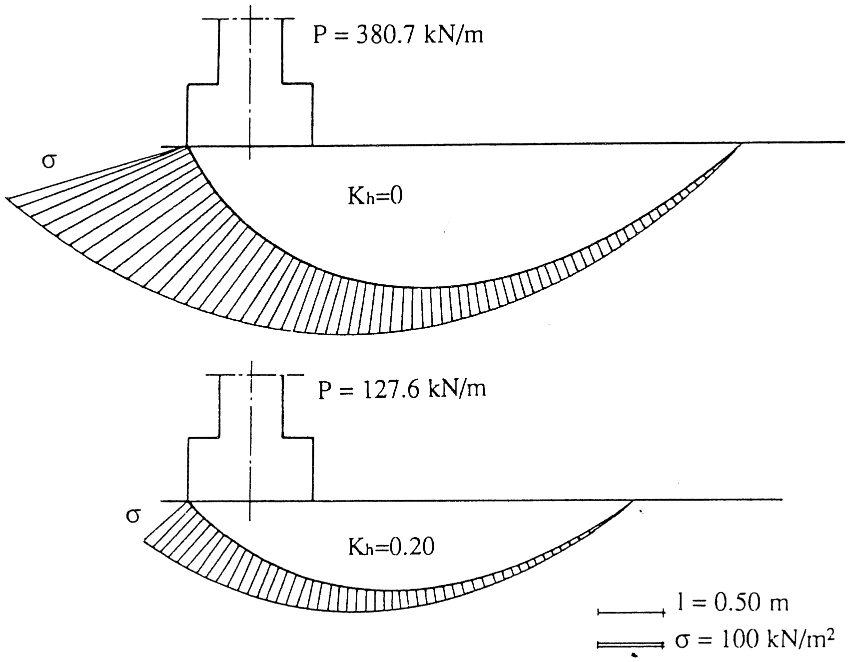

Fig. 5 : Critical slip surfaces and normal stress distributions

To investigate the influence of the seismic loadings on the normal stress distribution, we present in the same figure the influence of the k_h-values on the normal stress distribution in the same case mentioned above. This figure shows that due to the increase in the k_h-value, the normal stresses along the slip surface become smaller in the case of earthquake loadings.

CONCLUSION

A variational approach is applied to the limit equilibrium method to evaluate the seismic bearing capacity of a strip footing in a quasi-static manner. The approach presented is equivalent to the upper-bound method in limit analysis for a rotational log-spiral mechanism. There is good agreement with other author's results. This method is general and can be applied to complex geometry and various types of loadings.

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