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Influence of seepage flow on the passive earth pressures

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The variational limit equilibrium method is used for calculating the passive earth pressure coefficients in the presence of seepage flow. Numerical results are presented and discussed.

Introduction

When deep sheet piling structures are used in geotechnical engineering, we are frequently confronted by seepage flow around these structures as the excavation level is well below the groundwater table.

It is well known that the upward seepage forces will cause instability of the sheet piling structure. This is caused by the reduction of the passive earth force. Our aim in this paper is to propose an outline for the calculation of the passive earth pressures, taking into account the seepage forces.

Traditionally, the determination of these pressures is made using the classical method introduced by Terzaghi (ref.1). This Author has used an extension of Coulomb's sliding wedge theory in which seepage effects are taken into account by the addition of the pore water pressure due to the seepage flow on the plane sliding surface.

In this Paper, we present a more rational method which allows a rigorous solution of the earth pressure, taking into consideration the seepage forces. This method is based on a variational approach. It is a rigorous one in regard to the limit equilibrium method as it makes no assumptions with respect to the shape of the slip surface and the normal stress distribution along this surface.

The variational limit equilibrium method is equivalent to the upper-bound method in limit analysis for a rotational log-spiral mechanism (ref.2), hence the solution obtained is an upper-bound one for a rigid perfectly plastic material obeying Hill's maximal work principle.

Brief overview of the variational limit equilibrium method

The classical method introduced by Terzaghi (ref.1) is a limit equilibrium

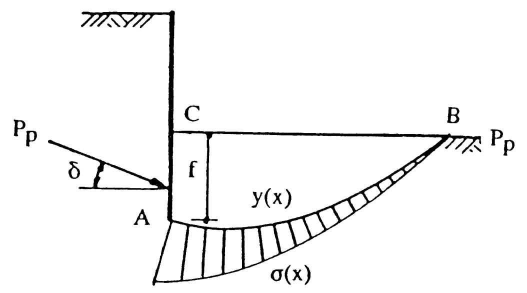

Fig.1. Slip surface and normal stress distribution for passive earth diagram pressure analysis

method giving unsafe solutions since it is based on Coulomb's approach which highly overestimates the passive earth pressure coefficients. This fact is due to the a priori hypothesis concerning the shape of the slip surface. In this paper, we look for the shape of the mechanism giving the minimum value of the passive earth force P_p, and for which the three limiting equilibrium equations are satisfied. This problem is formalized mathematically using a variational approach.

Mathematical formulation of the problem

It is well known that a rigorous limit equilibrium method is one for which the following conditions are satisfied.

(*a*) The shape of the slip surface y(x) and the normal stress distribution $\sigma_e(x)$ will give the minimum value of the passive earth force P_p.
(*b*) The three equations of the static equilibrium are satisfied for the soil mass ABC (Fig. 1).

Notice that a mass is in a state of limit equilibrium when the Mohr-Coulomb criterion is satisfied along the slip surface AB. Writing the three equations of the static equilibrium for the soil mass (Fig. 2), and combining these equations with the Mohr-Coulomb criterion, one obtains the three limiting equilibrium equations as follows

$$P_p.\cos\delta+U_1 = \int_{x_0}^{x_1}[\sigma_e(tg\phi+y')+u.y']\,dx \tag{1a}$$

$$P_p.\sin\delta= \int_{x_0}^{x_1}[\sigma_e(1-tg\phi y')+u-\gamma(f-y)]\,dx \tag{1b}$$

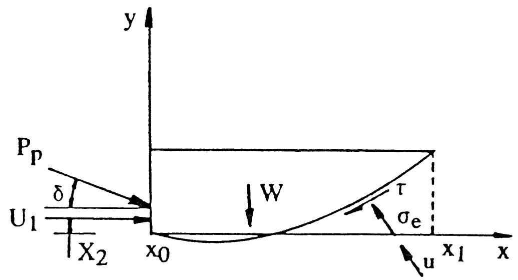

Fig. 2. Free body diagram

$$P_p.\cos\delta.f/3+U_1.X_2 = \int_{x_0}^{x_1}[\sigma_e(1-tg\phi y')x-\gamma.x(f-y)+u.x. +\sigma_e(tg\phi+y')y+u.y.y']\,dx \tag{1c}$$

where u represents the pore water pressure along y(x) and U_1 represents the resultant of the pore water pressure along the penetration depth f. A11 other parameters of these equations are defined in Fig. 2.

From these equations, it is easy to see that the passive earth force P_p is a functional of two functions y(x) and $\sigma_e(x)$. Thus, the rigorous passive limit equilibrium problem is a variational one of the isoperimetric type as follows

$$\text{Min } P_p = \int_{x_0}^{x_1}F(x,\ y,\ y',\sigma_e)\,dx$$

subject to

$$\int_{x_0}^{x_1}G_i(x,\ y,\ y',\ \sigma_e)\,dx = a_i \qquad (i=1,2)$$

where $F(x, y, y', \sigma_e)$ may be simply obtained through one of the equations (1). $G_i(x, y, y', \sigma_e)$ and a_i can be obtained from the two remaining equilibrium equations. It was shown (Petrov, ref.3) that the solution of such a problem is obtained using the Euler equations as follows

$$\frac{\partial H}{\partial \sigma_e} = \frac{d}{dx}\frac{\partial H}{\partial \sigma_e'} \tag{2a}$$

$$\frac{\partial H}{\partial y} = \frac{d}{dx}\frac{\partial H}{\partial y'} \tag{2b}$$

Where H is an intermediate functional given by

$$H = F + \lambda_i . G_i \qquad (i = 1,2)$$

Notice that H can be written as

$$H = \sigma_e . f(x, y, y') + g(x, y, y') \qquad (3)$$

Hence, equation (2a) is equivalent to f(x, y, y')=0. Solving this equation, one obtains the equation of the slip surface which is a log-spiral in the case of a constant ϕ (Fig. 4). Replacing this equation into equation (3), one can see that H becomes independent of the normal stress distribution. This result is a direct consequence of the shape of the slip surface. It was shown (ref.2) that any equation of the normal stress distribution having at least two degrees of freedom can satisfy the three equations of static equilibrium, and that only the equation of moments around the centre of the log-spiral is sufficient to calculate the passive earth force P_p. It is easy to see that the moment equation of the rotational log-spiral mechanism around the centre is identical to the work equation for the same mechanism in the upper-bound method in limit analysis. Thus, solving the passive earth pressure problem by writing the moment equation around the center of the log-spiral will give an upper-bound solution of the exact solution for an associated flow rule material. This method is detailed in the following section.

Calculation scheme of the passive earth pressures

As mentioned in the preceding section, the solution of the passive earth pressure problem by a variational limit equilibrium method is equivalent to saying that the equation of moment equilibrium must be satisfied for the soil mass bounded by the log-spiral AB and the ground surface BC. Notice that

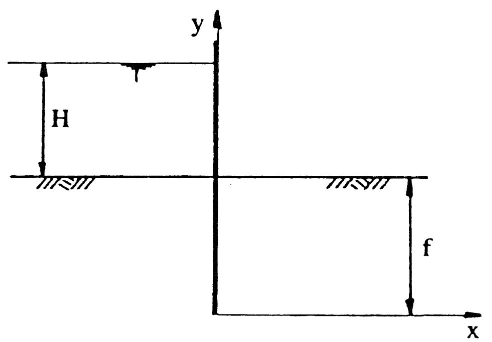

Fig.3. Illustrative example

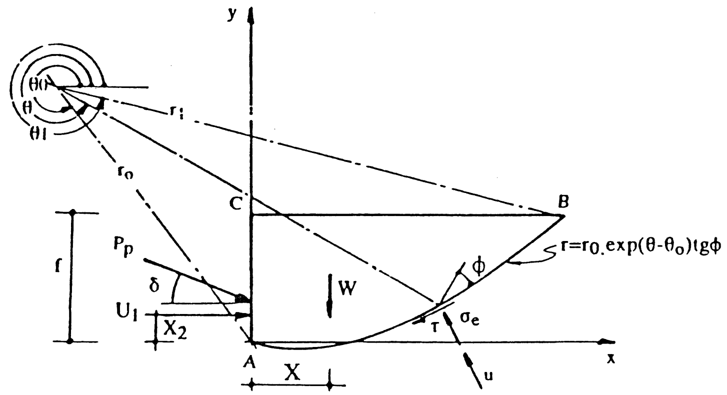

Fig.4. Log-spiral mechanism for passive earth pressure analysis

this conclusion is valid for any pore water pressure distribution in the soil mass. To investigate how the hydraulic head can affect the passive earth pressure coefficients, we present in the following section an illustrative example.

Illustrative example

As shown in Fig. 3 we consider a sheet pile whose penetration depth is equal to f and subject to a hydraulic head H. The soil is assumed to be homogeneous. To calculate the passive earth force, one must verify the equation of the moment equilibrium of the soil mass around the log-spiral centre. The external forces acting on the soil mass are shown on the free body diagram shown in Fig. 4. These forces consist of

(*a*) the weight of the soil mass between the log-spiral surface and the ground surface.
(*b*) the passive earth force which is inclined at δ to the normal of the sheet piling wall.
(*c*) the pore water pressures along the penetration depth and the log-spiral surface.
(*d*) the normal and tangential stress distributions along y(x).

Finally, the moment equation can be written as follows

$$W(r_0 . \cos\theta_0 + X) + P_p . \sin\delta (r_0 . \cos\theta_0) = P_p . \cos\delta (-r_0 . \sin\theta_0 - \frac{f}{3}) + M_1 + M_2$$

where M_1 and M_2 represent the moments of the force U_1 and of the pore water pressure along y(x) respectively.

Details concerning the calculation of moments of the weight and the passive earth force are given elsewhere (ref.2). We will proceed now to the calculation of the terms M_1 and M_2.

Calculation of the moments of the pore water pressures

It is well known that the pore water pressure distribution in the soil mass can be obtained analytically in this case. The relation which governs the hydraulic head h at any point (x, y) in the coordinate system (ox, oy) shown in Fig. 3 is given by

$$\frac{(f-y)^2}{f^2.\cos^2\Phi} - \frac{x^2}{f^2.\sin^2\Phi} = 1 \qquad (4)$$

where

$$\Phi = \pi.\left(\frac{h_s}{H} - \frac{1}{2}\right)$$

and

$$h = h_s + f$$

M_1-value (along the penetration depth)

Based on equation (4), one can easily show that along the penetration depth (in the downstream of the sheet pile), the resultant of the pore water pressures is given as

$$U_1 = \frac{\gamma_w.f^2}{2} + \gamma_w.H.f\left(\frac{1}{2} - \frac{1}{\pi}\right)$$

This force acts at a distance Y away from the point O. Y is given as

$$Y = \left[\frac{H\left(\frac{3}{8} - \frac{1}{\pi}\right) + \frac{f}{6}}{H\left(\frac{1}{2} - \frac{1}{\pi}\right) + \frac{f}{2}}\right].f$$

Finally, the moment of the force U_1 is given as $U_1\,(-r_o \sin q_o - Y)$.

M_2-value (along the slip surface)

This moment can be obtained by first calculating the elementary moment acting on a surface element ds and then by integrating along the surface AB as follows

$$M_2 = \int dM_2 = \int_{\theta_0}^{\theta_1} (u.\sin\phi).\frac{r.d\theta}{\cos\phi}.r = tg\phi.\int_{\theta_0}^{\theta_1} u.r^2.d\theta$$

Substituting the equation of the log-spiral in this equation, one obtains

$$M_2 = tg\phi.r_0^2.\exp(-2\theta_0.tg\phi).\int_{\theta_0}^{\theta_1} u.\exp(2\theta.tg\phi).d\theta$$

where $u = (h-y)\gamma_w = (h_s+f-y)\gamma_w$.

Finally, the moment M_2 can be written as

$$M_2 = tg\phi.r_0^2.\exp(-2\theta_0.tg\phi).\gamma_w.I$$

where $I = \int_{\theta_0}^{\theta_1} h_s.\exp(2\theta.tg\phi)\,d\theta + \int_{\theta_0}^{\theta_1} f.\exp(2\theta.tg\phi)\,d\theta - \int_{\theta_0}^{\theta_1} y.\exp(2\theta.tg\phi)\,d\theta = I_1+I_2+I_3$

While the integrals I_2 and I_3 are simples and can be obtained analytically, the integral I_3 however, cannot be obtained analytically and it needs numerical calculations (ref.2). Notice that this integral was calculated numerically using the gaussian quadrature method. The classical numerical methods (trapezoidal method) give insufficient accuracy in this case.

Minimization procedure

By writing the equation of the moment equilibrium around the centre of the log-spiral mechanism, one gets an expression of the passive earth force in terms of the θ_o and θ_1 angles. The most critical P_p-value is obtained by minimizing with respect to θ_o and θ_1 angles. The θ_o and θ_1 angles at which the P_p-value is minimum determine the most critical sliding surface. A

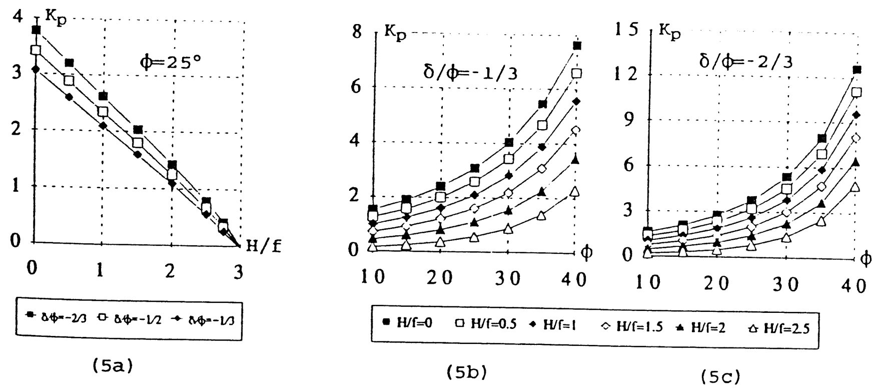

Fig.5. Influence of the hydraulic head on the K_p values

FORTRAN computer program has been developed for assessing the passive earth force and the corresponding critical slip surface.

Numerical results

Influence of the hydraulic head on the passive earth pressure coefficients

It is known that the seepage flow has the unfavourable effect of producing piping or heaving at the downstream of the sheet piling structure. Usually, when there is seepage flow, the sheet pile failure is considered to have taken place for high hydraulic head by heaving. The calculation of heaving risk is traditionally made by considering a rectangular mechanism adjacent to the sheet piling wall. This mechanism has a width equal to f/2. The vertical force equilibrium equation of this soil mass is then considered by neglecting the soil-structure force represented by the passive earth force. Kastner (ref.4), based on laboratory model tests, has shown that failure of the sheet piling structures is not only due to the heaving phenomena but it can be produced by reduction of the passive earth pressures. We present in Fig.5(a) one chart showing the influence of the hydraulic head on the passive earth pressure coefficient for an angle of internal friction ϕ equal to 25° and for three different values of δ/ϕ ($\delta/\phi = -1/3; -1/2; -2/3$).

It is easy to see that for zero K_p the corresponding H/f value is the same for different δ/ϕ values. This means that the angle of friction at the soil-structure interface has no effect on the H/f value causing failure by heaving. This fact can be explained as follows. When the passive earth force is vanishing, there is no interaction at the soil-structure interface and we have the traditional heaving phenomena. However, for small values of H/f for which the passive earth force does not vanish, this passive earth force depends on the δ value.

We present in Figs 5(b) and (c) some charts showing the variation of the k_p-value as a function of the nondimensional parameter H/f when $\gamma_{sat}/\gamma_w=2$ and for $\delta/\phi=-1/3, -2/3$. It is easy to see that k_p decreases with the H/f increase. The reduction is about 62% when H/f increases from zero to 2.5 for $\phi=40°$, $\delta/\phi=-2/3$.

Critical slip surface

The pore water pressures generated by seepage not only imposes extra loading to a soil mass but also shifts the sliding surface to less favourable positions. Consequently, in addition to the change in the passive earth pressure, the most critical sliding surface is also altered. The numerical results have shown that the slip surface approaches a planar surface due to the increase in the hydraulic head. Fig. 6 shows some typical changes in the critical sliding surface as the result of hydraulic head.

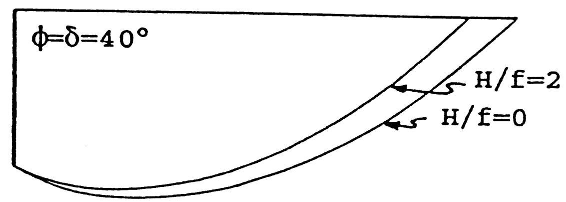

Fig.6. Critical slip surfaces

Analysis and discussion of results

Kastner (ref.4), based on laboratory model tests, has shown that for high hydraulic head, failure takes place for a planar slip surface. A previous theoretical work based on this hypothesis has been developed by Kastner et al (ref.5). The numerical results as obtained by the present variational approach are compared with the above mentioned results in the case ($\phi=25°$, $\delta/\phi=-2/3$) as shown in Fig. 7. As is well known, the planar slip surface greatly overestimates the k_p-values especially for high ϕ and δ values. Hence, for small hydraulic head corresponding to high passive force, the triangular mechanism gives greater k_p-values than the log-spiral one. The difference in the k_p-value for the two mechanisms decreases with the H/f increase. For a

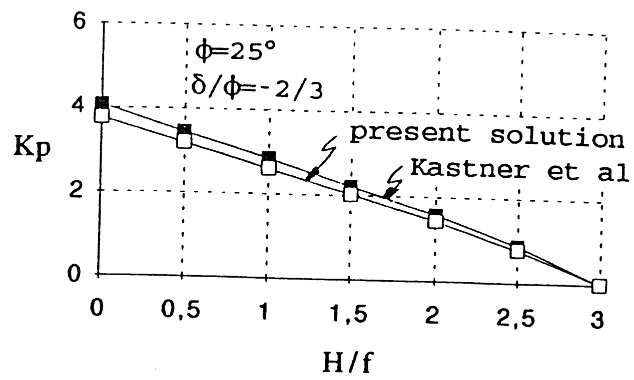

Fig.7. Comparison with the triangular mechanism (Kastner et al. 1987)

zero passive force corresponding to a failure by heaving, the two mechanisms give the same H/f value. This result confirms laboratory model tests of Marsland (ref.6) and Kastner (ref.4).

Conclusion

The variational limit equilibrium method is used to calculate the passive earth pressure coefficients in the presence of seepage flow. The numerical results as obtained by Kastner et al. (ref.5) concerning a planar slip surface are compared with the ones obtained with the above-mentioned approach. The present approach gives better solutions since we obtain less favourable mechanism giving less k_p-values.

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Discussion

SESSION 1

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During diaphragm wall construction under bentonite, the process of excavating a slot in the ground and back filling it with bentonite and subsequently with concrete will alter the lateral stress in the ground. For a given soil stratum and panel dimensions, the change of lateral stress in the soil adjacent to the diaphragm wall is governed mainly by the initial lateral stress in the ground and the lateral pressure exerted by concrete during construction. [1] Therefore, correct assessment of concrete pressure is paramount for modelling the wall installation process.

Full wet concrete pressure has often been assumed in numerical analysis along the entire depth of wall. [2,3] This assumption is contrary to theoretical considerations and the field evidence given below.

Theoretical consideration of lateral concrete pressure

According to extensive concrete research on the lateral pressure exerted by fresh concrete on vertical formwork, [4] it is widely accepted that the envelope of the lateral pressure with depth exerted by wet concrete is hydrostatic up to a certain limiting value (P_{max}) and thereafter remains constant with depth as shown in Fig. 1. Full fluid concrete pressure will not develop over the whole depth of the formwork since the concrete continues to set and harden as concreting progresses. The maximum pressure (P_{max}) on vertical formwork is governed by a number of factors such as the rate of placing, cross-section and depth of the form, temperature and concrete mix. The critical depth, h_c, below which the horizontal pressure is constant can be determined once the P_{max} value is known ($h_c = P_{max}/\gamma_c$) where γ_c is the unit weight of wet concrete. CIRIA report 108 [4] provides general guidance for calculating the P_{max} value.

There are two major differences between concrete placed in vertical formwork and concrete placed under bentonite in a diaphragm wall trench. Firstly, the pressure increase due to concreting depends on the difference between the unit weights of the concrete (γ_c) and bentonite (γ_b) rather than the unit weight of concrete alone. Secondly, the envelope of lateral pressure with depth increases beneath the critical depth at a slope governed by the unit weight of the bentonite, rather than staying constant. Thus a theoretical