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# DESIGN CHARTS FOR THE SEISMIC BEARING CAPACITY OF STRIP FOOTINGS ON SLOPES.

Abdul-Hamid SOUBRA* and Franck REYNOLDS**

**Abstract:** The seismic bearing capacity of a strip footing located on the top of a slope is calculated. The approach used is pseudo-static. The calculation scheme is based on the upper-bound method in limit analysis for a rotational log-spiral mechanism. Some charts are presented and discussed.

**Résumé:** On présente dans cet article le calcul de la capacité portante d'une fondation superficielle en tête de talus en présence de séisme. L'approche proposée est de type pseudo-statique et le schéma de calcul est basé sur la méthode de la borne supérieure en analyse limite pour un mécanisme rotationnel en spirale logarithmique. Des abaques sont présentés et commentés.

## 1 Introduction

A major consideration in the design of a strip footing in seismic areas is its stability against catastrophic failure. This problem of the ultimate bearing capacity of shallow strip footings under earthquake loadings has not been studied in detail in the past.

The very little studies available in literature describing the seismic effect on the bearing capacity of foundations concern those of Meyerhof [1] and Shinohara et al [2]. Both the approaches are pseudo-static where horizontal and vertical accelerations are applied to the centre of gravity of the structure and the problem is reduced to a static case of bearing capacity with inclined eccentric loads. However, in these solutions, the inertia of the soil mass is not included.

Recently, Sarma and Iossifelis [3] have considered this problem of the bearing capacity of strip footings in seismic areas by considering the inertial forces on all parts of the soil-foundation system. However, their approach is based on a limit equilibrium method making assumptions concerning the slip surface and the distribution of the inter-slice forces.

In this paper, the inertial forces are considered on all parts of the soil-foundation system and the method used is a variational limit equilibrium method: This method does not make assumptions concerning the slip surface and the normal stress distribution along this surface. It is also an upper-bound method in limit analysis

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for a rotational log-spiral mechanism. Hence, the solution obtained is an upper-bound one for an associated flow rule material obeying Hill's maximal work principle.

## 2 Theory

The following assumptions have been made in the analysis:

a. As was mentioned before, all inertias of the soil-structure system are considered in this analysis.

b. The earthquake acceleration for both the soil and the structure is assumed to be the same: Only the horizontal seismic coefficient $k_h$ is considered in this analysis, the vertical seismic coefficient is often disregarded.

c. The earthquake load on the structure is represented by the base shear load acting at the foundation level and an eccentricity for the vertical foundation load.

d. A one sided failure is assumed to occur along the surface y(x) (fig. 1).

e. An infinite slope is considered in this analysis where the slip surface y(x) intersects the slope at point C. However, for a relatively small fill, the base failure might be predominant and little modification in calculation is necessarily.

f. Only the reduction of the bearing capacity due to the increase in driving forces is investigated under seismic loading conditions. The shear strength of the soil is assumed to remain unaffected by the seismic loading.

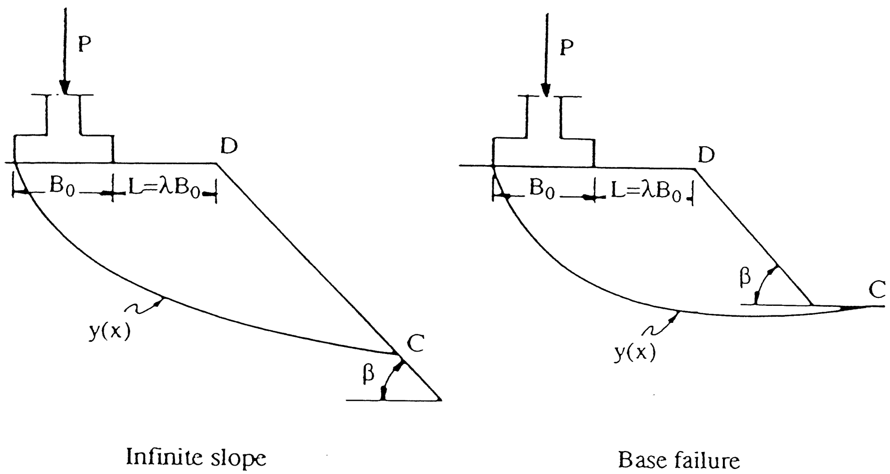



**Fig. 1 :** Slip surface for seismic bearing capacity analysis

As it is generally known, the traditional limit equilibrium methods give approximate solutions for the stability problems in geotechnical engineering. This fact is due to the a priori hypothesis concerning the shape of the slip surface and the



normal stress distribution along this surface. In this paper we briefly describe a variational limit equilibrium method. This method allows us to determine the shape of the mechanism giving the minimum value of the seismic bearing capacity and for which the three limiting equilibrium equations are satisfied. For further details concerning this approach, refer to previous works of Soubra [4].

### 2.1 LIMITING EQUILIBRIUM EQUATIONS

As shown (Fig. 1), the strip footing rests on the top of a slope of inclination β, apart from the slope shoulder D by a distance $L=\lambda.B_0$ where $B_0$ is the breadth of the footing. The analysis considers the case of a surface footing without any surcharge pressure.

Writing the three equations of the static equilibrium for the soil mass (Fig. 2) and combining these equations with the Mohr-Coulomb criterion, one obtains the three limiting equilibrium equations as follows

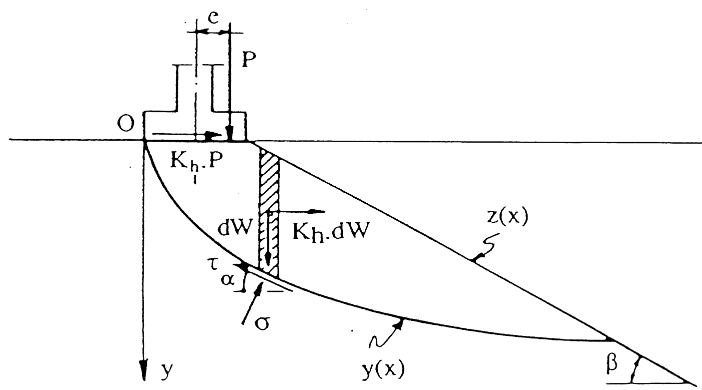



**Fig. 2 :** Free body diagram

$$P-\int_0^{x_1}[(c+\sigma tg\phi)y'+\sigma-\gamma(y-z)]dx=0 \qquad (1)$$

$$\int_0^{x_1}[\sigma y'-(c+\sigma tg\phi)+k_h\gamma(y-z)]dx\ +k_hP=0 \qquad (2)$$

$$\int_0^{x_1}[c+\sigma(tg\phi-y')]y-[(c+\sigma tg\phi)y'+\sigma]x-\frac{k_h}{2}\gamma(y^2-z^2)+\gamma x(y-z)dx$$

$$+P(\frac{B_0}{2}+e)=0 \qquad (3)$$

where equations (1) and (2) represent the equilibrium equations in the y and x directions respectively and equation (3) represent the moment equation around point O.

From these equations, it is easy to see that the foundation load P is a functional of two functions y(x) and σ(x). Thus, the rigourous bearing capacity problem is a variational one of the isoperimetric type as follows

$$\text{Min } P=\int_0^{x1}F(x,\ y,\ y',\sigma)dx$$

subject to

$$\int_0^{x_1}G_i(x,\ y,\ y',\sigma)dx=a_i \qquad (i=1,2)$$

where F(x, y, y', σ) is simply obtained through one of the equations (1, 2, 3). $G_i(x, y, y', \sigma)$ and $a_i$ can be obtained from the two remaining equilibrium equations. It was shown by Petrov [5] that the solution of such a problem is obtained using the Euler equations as follows

$$\frac{\partial H}{\partial \sigma}=\frac{d}{dx}\frac{\partial H}{\partial \sigma'} \qquad (4)$$

$$\frac{\partial H}{\partial y}=\frac{d}{dx}\frac{\partial H}{\partial y'} \qquad (5)$$

where H is an intermediate functional given by

$$H=F+\lambda_i.G_i \qquad (6)$$



Notice that H is linear in σ, hence equation (4) is equivalent to f(x, y, y')=0. Solving this equation, one obtains the equation of the slip surface which is a log-spiral in the case of a constant φ. Replacing this equation into equation (6), one can see that H becomes independent of the normal stress distribution. This result is a direct consequence of the shape of the slip surface. It was shown by Soubra [4] that any equation of the normal stress distribution having at least two degrees of freedom will satisfy the three equations of static equilibrium and that only the equation of moments around the centre of the log-spiral is sufficient to calculate the foundation load P. It is easy to see that the moment equation of the rotational log-spiral mechanism around the centre is identical to the work equation for the same mechanism in the upper-bound method in limit analysis. Thus, solving the bearing capacity problem by the upper-bound method in limit analysis for a rotational log-spiral mechanism will give a rigourous solution in regard to the limit equilibrium method. This method is detailed in the following section

## 2.2 UPPER-BOUND METHOD

According to the upper-bound theorem in limit analysis, for a kinematically admissible mechanism, the rate of external work exceeds the internal rate of dissipation of energy along the plastically deformed region. Thus, equating the external rate of work of all external forces to the internal rate of dissipation of energy gives an upper-bound of the exact solution for an associated flow rule material.

As was mentioned in the preceeding section, the mechanism considered in this paper consists of a rotational log-spiral mechanism whose equation is given as

$$r=r_0.\exp(\theta-\theta_0)\tan\phi$$

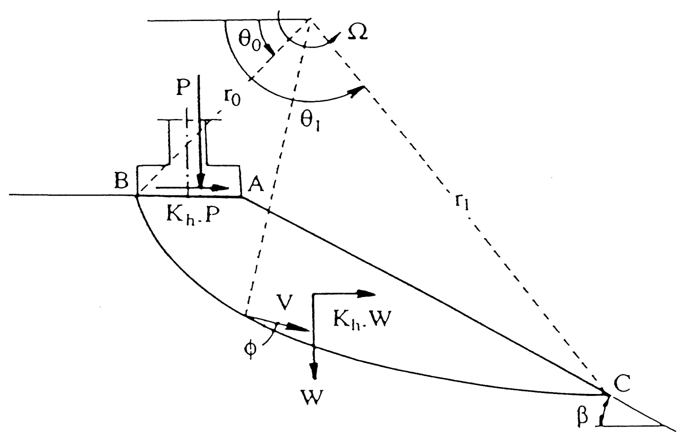



**Fig. 3 :** Rotational log-spiral mechanism for seismic bearing capacity analysis

For a rigid body rotation, this mechanism is kinematically admissible since the velocity V along the plastically deformed surface BC makes an angle $\phi$ with the transition layer according to the normality condition for an associated flow rule material.

### 2.2.1 Calculations of incremental external work

The incremental external work due to an external force is the external force multiplied by the corresponding incremental displacement or velocity. As shown (Fig. 3), the external forces contributing in the incremental external work consist of:

-The foundation load
-The weight of the soil mass
-The inertial forces on:
a.The soil-structure interface $K_h.P$
b.The soil mass $K_h.W$

The currently used values of $K_h$ ly between 0.05 and 0.15 in the United States and between 0.15 and 0.25 in Japan. Notice that the choice of the seismic coefficient is completely emperical (Seed [6,7,8]).

The incremental external work for the different external forces can be easly obtained as follows:

a. Incremental external work due to self weight and inertia of the soil mass ABC.

$$\Delta W_{ABC}=\gamma.A\,(X_1+K_h.Y_1).\Omega \qquad (7)$$

b. Incremental external work due to the foundation load and the corresponding shear load:

$$\Delta W_P=P\,(X_2+K_hY_2).\Omega \qquad (8)$$

The total incremental external work is the summation of these two parts of contributions, equation (7) and (8). That is:

$$\Sigma[\Delta W]_{ext}=\Delta W_{ABC}+\Delta W_P \qquad (9)$$

### 2.2.2 Calculations of incremental energy dissipation

According to Finn [9], the incremental energy dissipation per unit length along a velocity discontinuity or a narrow transition zone can be expressed as follows

$$\Delta D_L=c.\Delta V.\cos\phi$$



It is noted that the internal dissipation is totally independent of the seismic coefficient. The energy dissipation takes place along surface BC, therefore;

Along BC, there is:

$$\Sigma[\Delta D]=\Delta D_{BC}=\int_{\theta_0}^{\theta_1} c.\frac{r.d\theta}{\cos\phi}.V\cos\phi \;=c.r_0^2.f_1(\theta_0,\theta_1).\Omega \qquad (10)$$

where $f_1(\theta_0,\theta_1)$ is given by Soubra [4]. For a cohesionless soil, this dissipation is vanishing.

By equating $\Sigma[\Delta W]_{ext}$ in equation (9) to $\Delta D_{BC}$ in equation (10), we have:

$$q_c=\frac{P}{B_0}=\gamma\frac{B_0}{2}.N_\gamma+c.N_c \qquad (11)$$

where $N_\gamma$ and $N_c$ are the seismic bearing capacity factors. The most critical $q_c$-value can be obtained by minimization with respect to $\theta_0$ and $\theta_1$ angles [Fig. 3].

A pascal computer program for assessing seismic bearing capacity coefficients has been developed with equation (11) as a basis. The program gives the critical angles corresponding to the critical bearing capacity .

## 3 Numerical results and discussions

As mentioned previously, the present limit analysis solutions are valid when there is no reduction in soil strength due to an earthquake. This is the case only when the seismic acceleration a<=0.3g since most soils will either liquify or seriously weaken if a>0.3g (Okamoto [10]).

### 3.1 BEARING CAPACITY FACTORS

The bearing capacity factors $N_\gamma$ and $N_c$ are presented (Fig. 4). It is evident from these figures that these factors follows the same trend with respect to $\phi$ and $\beta$.These factors increase with an increase in $\phi$ and decrease with an increase in $\beta$.

| $N_\gamma$ | Saran & Agarwal | Present solution | |
|---|---|---|---|
| | β=0 | β=0 | β=5 |
| φ=20 | 6,36 | 7,33 | 4,91 |
| φ=25 | – | 16,52 | 9,88 |
| φ=30 | 29,45 | 38,07 | 18,76 |

| $N_c$ | Saran & Agarwal | Present solution | |
|---|---|---|---|
| | β=0 | β=0 | β=5 |
| φ=20 | 17,71 | 17,39 | 15,96 |
| φ=25 | – | 25,02 | 22,58 |
| φ=30 | 37,24 | 37,60 | 33,30 |

**Figure 4 :** Bearing capacity coefficients $N_\gamma$ and $N_c$

### 3.2 COMPARISON OF BEARING CAPACITY FACTORS

Recently, Saran and Agarwal [11] have proposed a non-symetrical mechanism to study the bearing capacity of eccentrically obliquely loaded footing. Their mechanism consists of a triangular wedge under the footing, a radial shear zone and a passive Rankine zone. The approach used in their analysis is a limit equilibrium method. Figure 4 compare the $N_\gamma$ and $N_c$ values obtained from the proposed analysis and those obtained from the available Saran and Agarwal's solution concerning the case (β=0°).

These values concern the static case ($k_h$=0) and a footing subject to central vertical loads. Figure 4 indicates that the proposed $N_\gamma$-values compare reasonably with Saran and Agarwal's solution, however; the values of $N_c$ obtained from the present analysis are very close to those obtained by Saran and Agarwal. Notice that the numerical results presented in this paper do not consider the moment due to the seismic loading on the structure (i.e. e=0), and the λ-value is taken equal to zero.

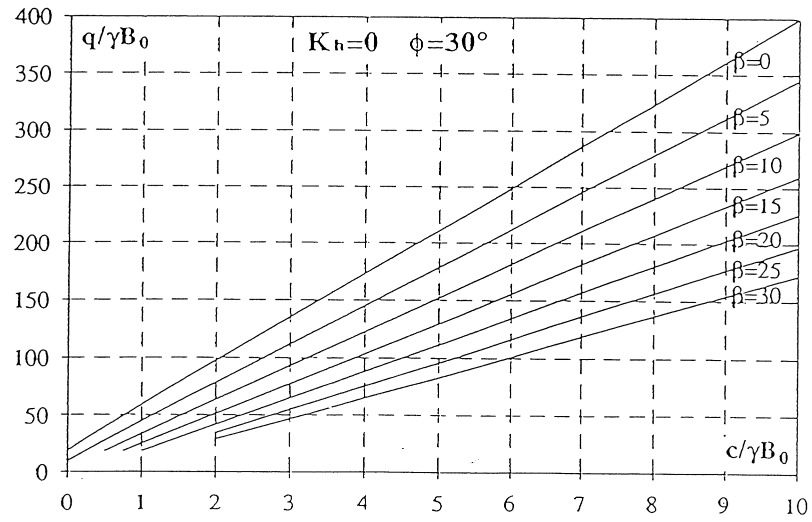
Kusakabe et al [12] have presented an upper-bound solution for the bearing capacity of foundations on slopes by introducing a failure mechanism which consists of a triangular active wedge, a log-spiral radial shear zone and a quadrilateral passive zone. For (φ=30°, $c/\gamma B_0$=1, β=15°), the value of $q/\gamma B_0$ as obtained by the present approach is equal to 25.05. The corresponding value given by Kusakabe et al and Bishop are respectively 28.9 and 29. This means that our upper-bound method gives interesting solution in this case. Notice that Bishop's method is a limit equilibrium method. His approach is based on a priori hypothesis. Hence, his solution is an approximate one and can not be classified as an upper-bound solution. Our method is more rigourous than the Bishop one since it is not based on a priori hypothesis and it verifies all the kinematical conditions of the problem.



Notice however that for large β-values, the coefficient $N_\gamma$ decreases very rapidly and becomes negatif due to the unfavorable effect of the soil mass. Notice also that the determination of the coefficients $N_\gamma$ and $N_c$ requires the minimization of each factors separetly: The rigourous solution consists in determining the total load P causing failure due to both weight and cohesion by unique minimization of the foundation load. For the two reasons mentioned above, we use in the following section the dimensionless parameters $q/\gamma B_0$ and $c/\gamma B_0$.

### 3.3 SOME DIMENSIONLESS CHARTS

To enable a general representation of results, the nondimensional parameters $q/\gamma B_0$ and $G=c/\gamma B_0$ are employed. Figures 5 indicate the variation of the $q/\gamma B_0$ values with the $c/\gamma B_0$ values for φ=30°, ($k_h$=0, 0.15) and for different values of slope angles β.



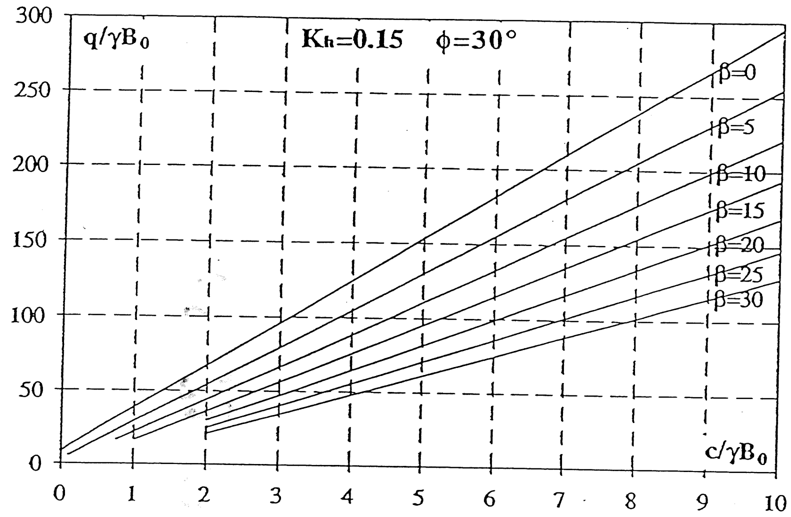



**Fig. 5 :** Variation of $q/\gamma B_0$ with $c/\gamma B_0$

Due to (Fig. 5), it is easy to see that for $\phi$=30°, $c/\gamma B_0$=10, $\beta$=30°, the reduction in the $q/\gamma B_0$ value is about 26% when the horizontal seismic coefficient increases form zero to 0.15. Thus, the calculation of the nondimensionalized bearing capacity $q/\gamma B_0$ taking into account the earthquake forces is of great interest in areas of high earthquake risks.

## 4 Conclusion

The upper-bound method in limit analysis for a rotational log-spiral mechanism is applied to the seismic bearing capacity problem for a strip footing resting on the top of a slope in a quasi-static manner. The approach presented is interesting since the solutions so obtained are in good agreement with the theoretical and experimental results of Saran and Agarwal in the case ($K_h = 0$) available in literature.

### Notations

| | |
|---|---|
| $k_h$ | horizontal seismic coefficient |
| $\sigma(x)$ | normal stress distribution along y(x) |
| $z(x)$ | equation of the ground surface |
| $x_1$ | abscissa of end point C |
| r | ray of the log-spiral mechanism |



| | |
|---|---|
| $r_0$, $\theta_0$ | initial ray and angle of the log-spiral mechanism |
| V | velocity |
| W | weight of the soil mass |
| $\Omega$ | angular velocity |
| A | area of the soil mass ABC |
| $X_1$, $Y_1$, $X_2$, $Y_2$ | distances from the point O to the line of action of forces W, $K_h$.W, P and $K_h$.P respectively. |
| $q_c$ | ultimate bearing pressure |
| a | seismic acceleration |

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