Passive Earth Pressure Coefficients in Seismic Areas by the Limit Analysis Method

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ABSTRACT

The upper-bound method in limit analysis is applied to the log-spiral rotational mechanism for calculating the passive earth presure coefficients in seismic areas. Numerical results are discussed and compared with other authors' results.

INTRODUCTION

Earthquake can endanger the stability of a soil-wall system by either increasing (or reducing) the active (passive) earth pressures acting on the wall. Thus, the dimensioning of deep sheet piling stuctures in seismic areas requires the determination of active and passive earth pressures acting on these structures taking into account the earthquake forces. So, a rational analysis of these pressures is of great interest in geotechnical engineering.

Traditionally, the determination of active earth pressures acting on a retaining wall and taking into account the earthquake forces, is made using the classical method introduced by Mononobe-Okabe [4]. In his method, this author used an extension of the Coulomb's sliding wedge theory [3] in which earthquake effects are taken into account by the addition of horizontal and vertical inertia terms.

In this paper, we present a more rational and simple method which makes it possible to calculate the earth pressure taking into consideration the earthquake forces. The approach presented is a rigorous one in regard to the limit equilibrium method since it makes no assumptions concerning the shape of the slip surface and the normal stress distribution along this surface.

It was shown by Soubra [15], that the variational limit equilibrium method is equivalent to the upper-bound method in limit analysis for a rotational mechanism. Hence, the solution obtained is an upper bound one for a rigid perfectly plastic material obeying Hill's maximal work principle.

VARIATIONAL LIMIT EQUILIBRIUM METHOD

The classical method introduced by Mononobe-Okabe [4] is a limit equilibrium method giving unsafe solutions since it is based on Coulomb's approach [3] which highly overestimates the passive earth pressure coefficients: This fact is due to the a priori hypothesis concerning the shape of the slip surface. In this paper, we look for the shape of the mechanism giving the minimum value of the passive earth force P_p and for which the three limiting equilibrium equations are satisfied. This problem is formalized mathematically using a variational approach.

Mathematical formulation of the problem
It is well known that a rigorous limit equilibrium method is one for which the following conditions are satisfied:

a. The shape of the slip surface y(x) and the normal stress distribution σ(x) will give the minimum value of the passive earth force P_p.
b. The three equations of the static equilibrium are satisfied for the soil mass ABC (fig. 1).

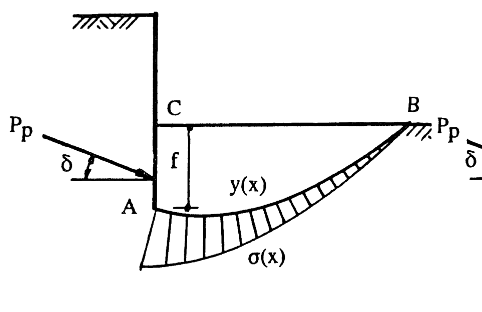


Figure 1. Slip surface and normal stress distribution for passive earth pressure analysis.

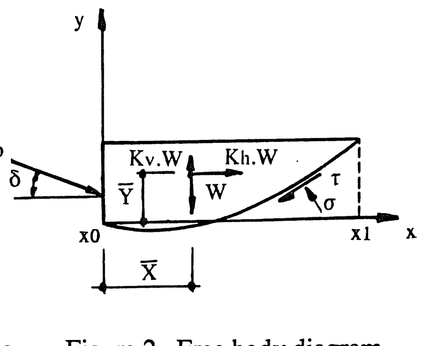


Figure 2. Free body diagram.

Notice that a mass is in a state of limit equilibrium when the Mohr-Coulomb criterion is satisfied along the slip surface AB. Writing the three equations of the static equilibrium for the soil mass (fig. 2), and combining these equations with the Mohr-Coulomb criterion; one obtains the three limiting equilibrium equations as follows

$$P_p.\cos\delta = \int_{x_0}^{x_1} [\sigma(tg\phi+y')-K_h.\gamma(f-y)]dx \qquad (1a)$$

$$P_p.\sin\delta = \int_{x_0}^{x_1} [\sigma(1-tg\phi.y')+\gamma(f-y)(K_v-1)]dx \qquad (1b)$$

$$P_p.\cos\delta.f/3 = \int_{x_0}^{x_1} [\sigma(1-tg\phi.y')x+\gamma.x(f-y)(K_v-1)-K_h.\gamma(f-y)y + \sigma(tg\phi+y')y]dx \qquad (1c)$$

where all parameters of these equations are defined in figure (2). From these equations, it is easy to see that the passive earth force P_p is a functional of two functions y(x) and σ(x). Thus, the rigourous passive limit equilibrium problem is a variational one of the isoperimetric type as follows

$$\text{Min } P_p = \int_{x_0}^{x_1} F(x, y, y', \sigma)dx$$

subject to

$$\int_{x_0}^{x_1} G_i(x, y, y', \sigma)dx = a_i \qquad (i=1,2)$$

where F(x, y, y', σ) is simply obtained through one of the equations (1). G_i(x, y, y', σ) and a_i can be obtained from the two remaining equilibrium equations. It was shown (Petrov [8]) that the solution of such a problem is obtained using the Euler equations as follows

$$\frac{\partial H}{\partial \sigma} = \frac{d}{dx}\frac{\partial H}{\partial \sigma'} \qquad (2a)$$

$$\frac{\partial H}{\partial y} = \frac{d}{dx}\frac{\partial H}{\partial y'} \qquad (2b)$$

Where H is an intermediate functional given by

$$H = F + \lambda_i.G_i \qquad (i= 1,2)$$

Notice that H can be written as

$$H = \sigma.f(x, y, y')+g(x, y, y') \quad (3)$$

Hence, equation (2a) is equivalent to : f(x, y, y')=0. Solving this equation, one obtains the equation of the slip surface which is a log-spiral in the case of a constant ϕ. Replacing this equation into equation (3), one can see that H becomes independent of the normal stress distribution. This result is a direct consequence of the shape of the slip surface. It was shown (Soubra [15]) that any equation of the normal stress distribution having at least two degrees of freedom will satisfy the three equations of static equilibrium and that, only the equation of moments around the centre of the log-spiral is sufficient to calculate the passive earth force P_p. It is easy to see that the moment equation of the rotational log-spiral mechanism around the centre is identical to the work equation for the same mechanism in the upper-bound method in limit analysis. Thus, solving the passive earth pressure problem by the upper-bound method in limit analysis for a rotational mechanism will give a rigorous solution in regard to the limit equilibrium method. This method is detailed in the following section.

UPPER-BOUND METHOD

The equation of the rotational log-spiral mechanism (fig. 3) is given as

$$r=r_0.e^{(\theta-\theta_0)tg\phi} \quad (4)$$

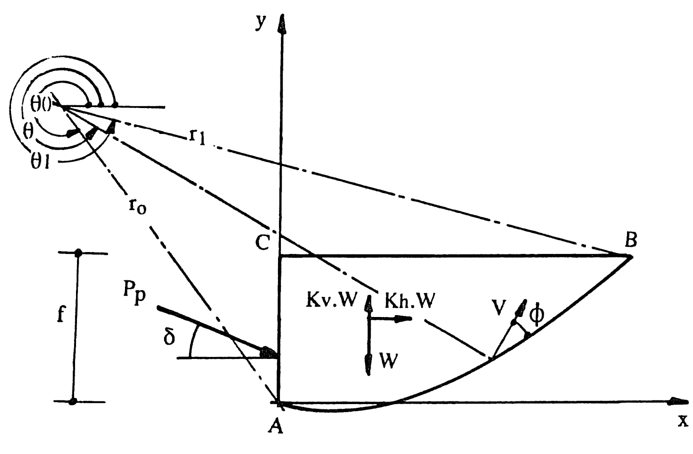


Figure3. Log-spiral mechanism for passive earth pressure analysis.

For a rigid body rotation, this mechanism is kinematically admissible since the velocity V along the plastically deformed surface AB (fig. 3) makes an angle ϕ with the transition layer according to the normality condition for an associated flow rule material.

According to the upper-bound theorem in limit analysis, for a kinematically admissible mechanism, the rate of external work exceeds the internal rate of dissipation of energy along the plastically deformed region. Thus, equating the external rate of work of all external forces to the internal rate of dissipation of energy gives an upper-bound of the exact solution for an associated flow rule material.

Rate of external work

The external forces acting on the soil mass are shown on the free body diagram shown in figure (3). These forces consist of:

a. The weight of the soil mass between the log-spiral surface and the ground surface.

b. The passive earth force which is inclined at δ to the normal of the sheet piling wall.

c. The force K.W which simulates the inertial force due to the earthquake effect.

Notice that the seismic vector K has two components: The horizontal seismic coefficient K_h whose value is dominating and the vertical seismic coefficient K_v which is often disregarded. The currently used values of K_h ly between 0.05 and 0.15 in the United States and between 0.15 and 0.25 in Japan. Notice that the choice of the seismic coefficient is completely empirical (Seed [11, 12, 13]). Seed [13] showed that a value of K_h which lies between 0.1 and 0.17 describes very well the failure of the Sheffield dam in California: This dam was subjected to a maximal base acceleration of 0.15g. This author has also shown that for higher accelerations (0.4-0.5g) which describe the Californian earthquakes, a minimum value of 0.3 is necessary for the horizontal seismic coefficient.

When studying the stability of slopes, Taniguchi and Sasaki [16] have analysed the failure which occured for a slope subjected to the Naganokon Seibu earthquake in 1984 in Japan. These authors have shown that the seismic coefficient can be described by either the following formulas

$$K_h = 0.65.\frac{a_{max}}{g} \qquad\qquad K_h = \frac{1}{3}.\left(\frac{a_{max}}{g}\right)^{1/3}$$

Finally, it is interesting to notice that the real value of the seismic coefficient requires the analysis of actual failure cases.

The weight of the soil mass ABC is given as

$$W = \int_{x_0}^{x_1} \gamma(f-y)dx \qquad (5)$$

where f is the penetration depth and y represents the equation of the slip surface in the coordinate system (ox, oy). Based on equation (4), one can easily show that

$$y = r.\sin\theta - r_0.\sin\theta_0$$
$$dx = r(tg\phi.\cos\theta - \sin\theta)d\theta$$

Replacing these equations into equation (5), it can be shown that

$$W = \gamma.r_0^2.f_1(\theta_0, \theta_1)$$

where $f1(\theta_0, \theta_1)$ is given elsewhere (Soubra [15]). Having established the weight of the soil mass, one can calculate the rate of external work done by the weight of the soil mass as the product of the weight by the vertical component of the velocity of the soil mass. The vertical component of the velocity is given as

$$V = \Omega(r_0.\cos\theta_0 + \overline{X})$$

where $\overline{X}$ represents the distance between the y axis and the line of action of the weight force, and Ω being the angular velocity of the soil mass. $\overline{X}$ is simply calculated as follows

$$\overline{X} = \frac{1}{S} \int_{x_0}^{x_1} x(f-y)dx = r_0.f_2(\theta_0, \theta_1)$$

where $f_2(\theta_0, \theta_1)$ is given elsewhere (Soubra [15]).

The rate of external work done by the passive earth force is given as

$$P_p[-r_0.\cos\theta_0.\sin\delta + \cos\delta(-r_0.\sin\theta_0 - \frac{f}{3})]\Omega$$

The rate of external work done by the horizontal inertial force $K_h.W$ is the product of this force by the horizontal velocity of the soil mass ABC as follows

$$K_h.W.\Omega(-r_0.\sin\theta_0 - \overline{Y})$$

where $\overline{Y}=r_0.f_3(\theta_0, \theta_1)$ and $f_3(\theta_0, \theta_1)$ is given by Soubra [15]

Rate of internal dissipation

The internal dissipation of energy along the log-spiral surface is simply calculated by first calculating the differential energy dissipation along AB which is the product of the surface element $r.d\theta/\cos\phi$ by the cohesion c by the tangential velocity $V\cos\phi$ and then by integrating over the surface AB as follows

$$D = \int_{\theta_0}^{\theta_1} c \frac{r d\theta}{\cos\phi} V\cos\phi$$

Replacing V by $\Omega.r$ and integrating, one obtains

$$D = c.r_0^2.\Omega.f_4(\theta_0,\theta_1)$$

where $f_4(\theta_0, \theta_1)$ is also given by Soubra [15]. For a cohesionless soil, this dissipation is vanishing.

Work equation

Equating the total external work done by the weight, the inertial force and the passive force P_p, to the internal rate of dissipation of energy, one gets

$$P_p = \frac{W[-(\overline{X}+r_0.\cos\theta_0)+K_h(-r_0.\sin\theta_0-\overline{Y})]}{r_0.\cos\theta_0.\sin\delta-\cos\delta(-r_0.\sin\theta_0-\frac{f}{3})} \qquad (6)$$

Notice here that the passive earth force P_p is assumed to act at the bottom third of the penetration depth. This hypothesis depends on problem kinematics and it will be discussed later. Due to this hypothesis, one can write $P_p= K_p.\gamma.f^2/2$. The most critical K_p-value can be obtained by minimizing with respect to θ_0 and θ_1 angles shown in figure (3). The θ_0 and θ_1 at which the K_p-value is minimum determine the most critical sliding surface. A FORTRAN computer program for assessing seismic passive earth pressures has been developed with equation (6) as a basis.

NUMERICAL RESULTS

Effect of the point of action of the passive earth force on the passive earth pressure coefficients

In fact, the point of action of the passive earth force depends greatly on problem kinematics. This point was the subject of great controversy in literature. Prakash and Basavanna [9] showed that the point of action of the active earth force lies between 0.4f and 0.5f when the seismic coefficient varies between 0.1 and 0.3. Wood [17] was based on the elastic soil hypothesis and suggested a force acting at the middle of wall height. Aubry and Chouvet [1] made a finite element analysis and suggested a point of action lying slightly higher than the bottom third of the wall height.The present analysis have shown that the passive earth pressure coefficient is increased when the passive force goes up. Thus, a conservative approach concerning the K_p-value is to adopt the bottom third distance.

Seismic effect on the passive earth pressure coefficients

It is known that earthquakes have the unfavorable effect of increasing active and decreasing passive lateral earth pressures. An earthquake can also reduce the shearing resistance of a soil. The reduction in the shearing resistance of a soil during an earthquake is only effective when the magnitude of the earthquake exceeds a certain limit and the ground conditions are favorable for such a reduction. The evaluation of such a reduction requires considerable knowledge in earthquake engineering and soil dynamics. Research conducted by Okamoto [7] indicated that when the average ground acceleration is larger than 0.3g, there is a considerable reduction in strength for most soils. However, he claimed that in many cases, the ground acceleration is less than 0.3g and the mechanical properties of most soils do not change significantly in these cases. In this paper, the shear strength of the soil is assumed to remain unaffected as the result of the seismic loading.

To investigate how the passive earth pressures are affected, numerical results based on the above mentioned upper-bound method in limit analysis for a rotational mechanism are presented in dimensionless form (figure 4). As mentioned previously, the present limit analysis solutions are valid when there is no reduction in soil strength due to an earthquake. Due to figure (4), it is easy to see that for $\phi=40$; $\delta/\phi=-2/3$; the reduction in the passive earth pressure coefficient is about 16.5% when the horizontal seismic coefficient increases from zero to 0.3. Thus, the calculation of the coefficients of passive earth pressure taking into account the earthquake forces is of great interest in areas of high earthquake risks.

Comparison with authors' results

The best upper-bound solution in limit analysis is given by Chang and Chen[2] for the translational log-sandwich mechanism. His results have shown that the Mononobe-Okabe approach seriously overestimates the K_p-value. This is especially the case when the wall is rough.

The results obtained by the present upper-bound method in limit analysis for a rotational log-spiral mechanism are compared with the above mentioned upper-bound solutions (figure 4). It is interesting to remember here that the log-sandwich translational mechanism is the best mechanism available in literature since it gives the smallest upper-bound solution. Our approach gives better solutions than the Chang and Chen log-sandwich ones since our passive earth pressure coefficients are smaller than those of Chang and Chen [2] for $\delta>0$. However, when $\delta=0$; we obtain a planar surface and our passive earth pressure coefficients are the same as those of Chang and Chen since both the log-spiral and the log-sandwich mechanisms degenerate to a planar surface when $\delta=0$.

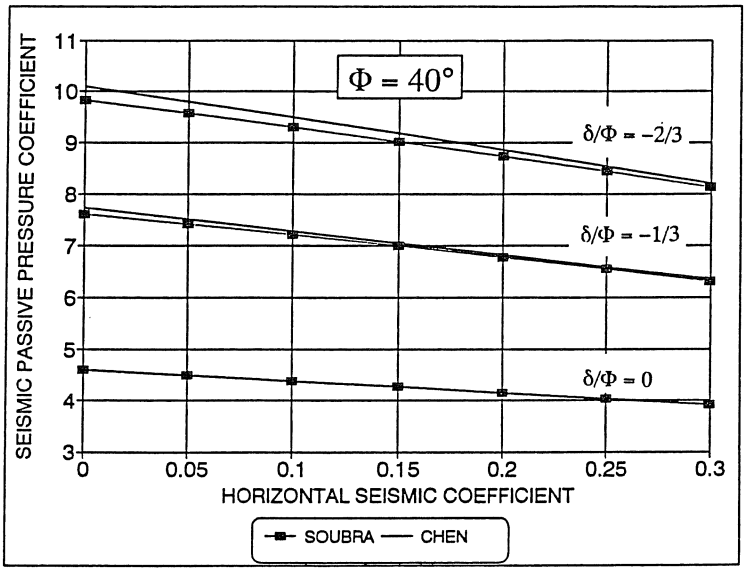


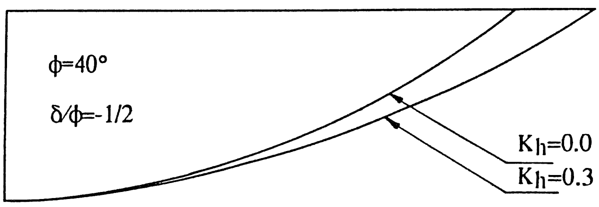
Figure 4. Some Kp-value by the present analysis and the Chang-Chen's one.

For $\delta>0$ ($\phi=40$, $\delta/\phi=-2/3$ for example); the passive earth pressure coefficient as calculated by the present approach is 3.7% smaller than the Chang and Chen's one when $K_h=0$.This difference decreases with the increasing of the K_h-value. This difference is about 0.9% when $K_h=0.3$.

In order to braket the collapse load, our solution is compared with the lower-bound solution for ($\phi=40$, $\delta/\phi=-1/2$) available in literature.This comparison shows that our upper-bound solution (K_p=9.81) is 2.8% greater than the Lysmer [5] lower bound solution (K_p=9.54) which indicates that the upper-bound solution in limit analysis for a rotational log-spiral mechanism is very close to the exact solution for an associated flow rule material.

Seismic effect on the critical slip surface

The seismic acceleration generated by earthquakes not only imposes extra loading to a soil mass but also shifts the sliding surface to less favorable positions. Consequently, in addition to the change in the passive earth pressures, the most critical sliding surface is also altered. The numerical results given by the Fortran computer program have shown that the slip surface approaches a planar surface due to the increase in the K_h-value in the case of a rough wall (δ>0). Whereas, in the case of a smooth wall (δ=0); when the K_h-value is equal to zero, the slip surface is planar making an angle equal to (π/4 – ϕ/2) with the horizontal direction: This is in accordance with the Rankine solution. For higher values of K_h, the slip surface remains planar, but it is inclined at smaller angles than the K_h=o case. Figure (5) shows some typical changes in the critical sliding surface as the result of an earthquake.



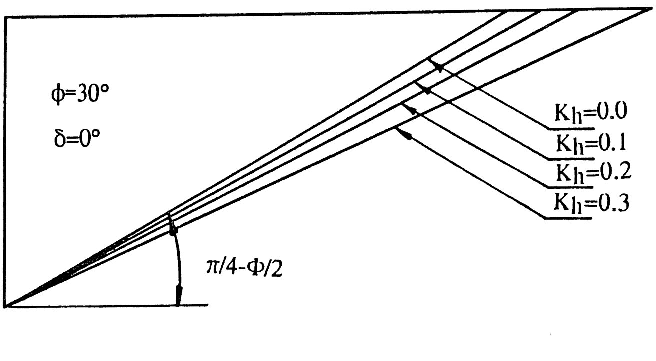


Figure 5. Effect of seismic forces on failure mechanism.

Finally, it is interesting to notice that the critical sliding surface becomes more extended when earthquakes occur. This conforms with the experimental results of Murphy [6]. The change in the critical sliding surface as the result of earthquake has also been noted by Sabzevari and Ghahramani [14].

CONCLUSION

The upper-bound technique of limit analysis for a rotational log-spiral mechanism is used for determining the seismic passive earth pressure coefficients in a quasi-static manner. The approach presented is interesting since the passive earth pressure coefficients so obtained are smaller than the ones given by the best upper-bound solution available in literature concerning the translational log-sandwich mechanism (Chang and Chen [2]) and the difference with the lower-bound solution (available only when K_h=0) is less than 3% in the (ϕ=40°;δ=20°) case.

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